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Combined Fourier hologram as a noise-resist method of synthesis 2D computer-generated hologram with single diffractive order

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Abstract. In this paper, a comparative analysis of two approaches to synthesis 2D hologram, which restores optical field without conjugated images, is performed. These holograms are used in embossed rainbow protective holograms to improve their forgery-proof characteristics. As a basis, one method of kinoform hologram synthesis is used, which allows to obtain the restored image of acceptable quality, but phase-matching condition is necessary to complete. That limits usage of this method in mass production of embossed holograms. The offered method of combined holograms is based on addition of optical fields restored by separated Fourier holograms. This method let us to avoid hard requirements to technical process on resist – polymer relief translating accuracy. Thus, the proposed method can be recommended for application in mass production of rainbow protective holograms.

Keywords: computer-generated hologram, kinoform, protective hologram.

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1. Introduction

Computer generated Fourier holograms (Fourier CGH) are widely used for recording the concealed images in holographic protective elements. Decoding these holograms is realized in optical way with use of coherent decoders. The offered combined hologram is the same in its principle. It allows strengthening the resistance of typical protective elements to forgery and increase an objectivity of their identification procedure without extra expenses on changing the decoding scheme.

One of the main problems that we face with when recording Fourier holograms is recording the complex value appearing in the frequency plane. Usually, phase information is simply omitted, and only the intensity distribution is used to record the Fourier hologram, so a final hologram doesn't contain enough information for full object wave reconstruction.

There are two fundamental different ways to solve this problem:

First: use of various methods of Fourier spectrum encoding that allows going from the complex quantity of field distribution in a hologram registration plane to the real quantity [1,2]. The methods in which a phase is encoded by means of space carrier are of the largest interest [2]. They combine more naturally with modern recording devices and media (typical resists for laser and E-beam lithographic systems). We call these holographic methods as optimized for digital synthesis.

Second fundamental way lies in recording the Fourier spectrum on double-layer amplitude-phase recording media [4-5]. This method is technologically possible but not suitable for integration of these structures into the holographic protective elements. However, it is possible to use its cut-down version when in the computed Fourier spectrum the amplitude is forcibly assumed to be equal to a constant and phase recording is realized, for example, by phase modulation of a geometric relief of the recording medium. The kinoform, for example, operated by using this principle [6].

It is necessary to mention that digital Fourier holograms are flat and act as flat 2D diffraction grating. Thus, they restore two diffraction orders – two conjugate images. It applies restriction on the recorded object – function **a** (x, y) must satisfy causality condition and turn into zero at $x < 0$:

$$a(x,y) = A(x,y)\cdot\exp(ij(x,y)) = 0 \quad \text{at } x < 0 \qquad (1)$$

Otherwise, mutual overlay of the real and conjugate images will occur.

This paper studies how to enlarge the possibilities of synthesis and recording the digital Fourier spectrums by removing the restrictions connected with causality condition.

From the practical viewpoint, it will allow to increase the resistance of the hologram to falsification (synthesis algorithm changes, demands to technological process and materials are increased), to increase intensity of the image restored (owing to energy re-distribution), to enlarge the possibilities for design solutions (owing to removing the restrictions) and to receive some extra features like as easy relief-depth laser control and restoring different images under light sources with different wavelengths.

Below, we examine two approaches: use of quadrature hologram that is proposed one and using the Zero-Order Device – kinoform.

2. Theory

2.1. Elimination of conjugated images by using combined holograms

When recording a traditional Fourier hologram on recording medium, in accordance with a computed Interference Fringe Data (IFD) an intensity distribution is recorded [7]:

$$I(x) = \left|1 + \exp(-i\,\omega_0\,x)\right|^2 = 2\cdot(1 + \cos(\omega_0\,x)) \qquad (2)$$

When restoring this interferogram falling normally with a plane wave, two images of the object appears – a conjugated image and the real one. Physical reason of twin-image appearance is explained by the fact that this hologram has no information about the direction from which the object wave felt on the hologram (in contrast, for example, to Denisyuk thicklayer hologram where direction of wave distribution is registered in the medium).

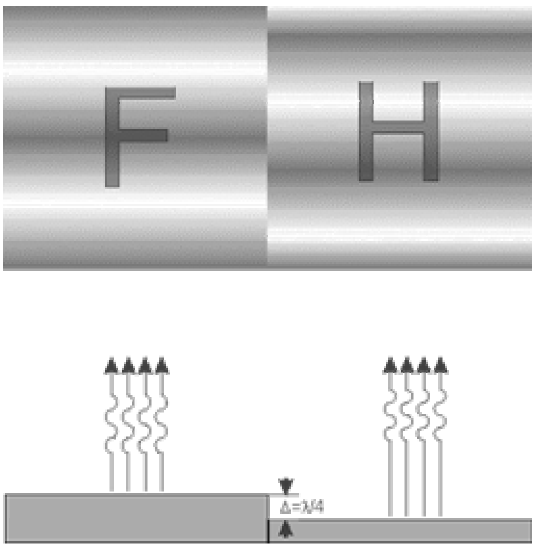

Fig. 1. Mutual disposition of elements in the combined hologram.

From the mathematical point of view, appearance of two conjugated waves at the stage of hologram restoring (instead of one when recording) is described by $\cos(\omega_0\,x)$ function parity that is integral part of the registered value. It is not possible to determine a growth direction of complex components $\exp(+i\,\omega_0\,x)$ and $\exp(-i\,\omega_0\,x)$, IFD for cases when object wave falling at θ and -θ angles will be the same.

It is logically to suppose that, if to insert an optical element in this recording scheme, which would shift a phase of the reference wave by $\frac{\pi}{2}$ radians, the same hologram but contained already the sine (odd) function appeared.

This hologram, where the intensity distribution is described by $\sin(\omega_0\,x)$ function is called the *quadrature* hologram. The intensity distribution in this hologram is described by the following expression:

$$I(x) = \left|\exp(\,i\,\pi/\,2) + \exp(\,\pm i\,\omega_0\,x)\right|^2 = \\ = 2(1 + \sin(\,\omega_0\,x)) \qquad (3)$$

To restore both holograms [8-10] (traditional and quadrature) are used, the wave phase restored by the quadrature hologram must be additionally shifted by λ/4, thus, the total wave field will be as follows:

$$u_r(x) = (1 + \cos\omega_0 x) + i(1 \pm \sin\omega_0 x) = \\ = (1 + i) + \exp(\pm\omega_0 x) \qquad (4)$$

As indicated in (4), when restoring, only one diffraction order remains. Combination of the traditional and quadrature holograms allows registering various intensity distributions in the hologram plane, in dependence on mutual source location of the object and reference waves.

Fig. 2. Images restored from Fourier hologram (a), Hilbert hologram (b) and combined hologram (c).

2.2 Use kinoform method for twin-image elimination

To evaluate advantages and disadvantages of the method proposed above, another hologram without conjugated orders was calculated by more typical way – using so-called kinoform. As mentioned above, kinoform [6, 11, 12] represents a special type of a thin-layer hologram, where a complex amplitude of an object light wave $a(x, y) = A(x, y)\cdot\exp[i\cdot(x, y)]$ in the plane of hologram registration is almost constant by its module. In practice, this situation takes place, when the object has a diffused surface, or lighted with a diffused light, or diffuser is installed on a way of the object wave distribution. In these cases, the image of the object can be restored using only the phase information $\varphi(x, y)$.

Synthesis of the kinoform is carried out according to the scheme of calculation of the Fourier hologram. The object is defined in the object plane as a set of point light sources. Then, after Fourier transformation, a mathematical analogue of the object wave in the hologram plane was calculated. As a basic assumption, a constancy of the module complex amplitude of the object wave $a(x, y) = \mathrm{const}\cdot\exp[i\cdot(x, y)]$ was made, in addition for losses minimization related to rejection of the amplitude information in the algorithm of calculations diffuser optimization was added. After calculation of a phase difference between the object and reference waves in every *(x,y)* point on the hologram plane, the result was normalized in such a manner that the phase function *(x, y)* varied within 0 to 2π range.

$$\tilde{\varphi}(x,y)\sum_{n=-N}^{N}\sum_{m=-M}^{M}\varphi_{nm}(\bmod 2\pi)\cdot\delta(x-n\Delta x, y-m\Delta y),$$
$$\varphi_{nm}(\bmod 2\pi) = \varphi - 2\pi j \qquad (5)$$
$$j\cdot 2\pi \le \varphi \le (j+1)\cdot 2\pi, \quad j = 0,1,2,3....$$

The received array of phase values is used for kinoform recording on the phase recording medium.

The quality of the image restored from the kinoform mainly depends on two factors - value constancies of the module in the complex amplitude object wave and a performance of a condition of the phase coordination (5). The performance of a condition of the phase matching depends on accuracy of the transfer of calculated discrete values of the phase onto the recording medium. The constancy of the amplitude value of the field in the plane of registration is provided owing to the insertion of the object diffuser. As a first approximation, the diffuser represents an array with random values of a phase (from 0 up to 2π).

This object-independent diffuser provides a rather constant object wave amplitude value in the field of the hologram registration, allows receiving the value of Signal to Noise Ratio (SNR) about 20 for the restored image. However, the granular structure of the image and a speckle around of the image remains. To eliminate these phenomena, an algorithm of diffuser optimization must be applied. Such algorithm was described in [13] and with some changes used for 3D CGH in [14, 15], we improved it in the next way: in the iterative cycle the initial array of the amplitude and phase values, which describes object is multiplied by the set of random phase values phase (diffuser).

Then, the Fourier transform from the object (actually, mathematically restored image) is calculated. Then SNR at this step of iteration is calculated. Value of SNR saved, then, the next iteration step is made. If after the next iteration, SNR value is higher than the saved one, value of a phase diffuser in the given point is accepted corresponding to maximal SNR value. The iterative cycle repeats itself or given number of times (or till achievement of the given SNR value). As a result, SNR value was increased up to 30. With the purpose to get more constant values of the amplitude complex field in the hologram plane, an additional emphasis of the object amplitude [15, 16] can be applied.

3. Experimental

We use a method of electron beam lithography for all CGH recordings on a phase recording medium (electron-sensitive PMMA resist) [17 - 19].

When any Multilevel CGH is recorded, the use of the methods of lithography digitization of the continuous function Δφ (*x, y*) is required. So, the calculated continuous function is replaced by a discrete function with the step $2\pi/L$, where L is a maximal possible number of levels of a phase relief in the given recording method.

$$t(x,y)\sim rect\left[\frac{x}{\Delta x}\right] rect\left[\frac{y}{\Delta y}\right]\exp\left[\pm ic\varphi(\bmod 2\pi)\right]\times$$
$$\times\sum_{n=-N}^{N}\sum_{m=-M}^{M}\delta(x-n\Delta x, y-m\Delta y) \qquad (6)$$

Where c is the factor that answers for 'phase matching' condition observance. When c = 1, the phase matching condition is observed, and conjugated orders in the reflected wave are eliminated. N, M – vertical/horizontal half-dimensions of the calculated IFD.

CCGH can be recorded as a typical Fourier hologram without any additional limitations posed on recording parameters. More complex task is for the

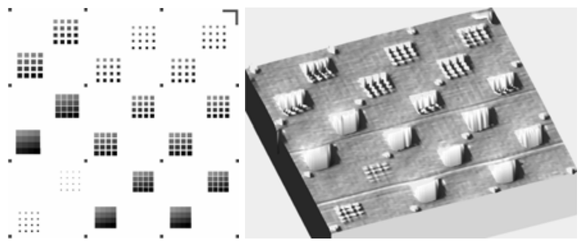
a b

Fig. 3. Test structure to determine optimum pixel size, exposure dose and off-duty factor value.

kinoform. From consideration of the transmission function for the kinoform, it follows that for restoration of the initial wave front without distortions that the constant *c* is required to be equaled to unity. It means that the light falling on the hologram's sample with a phase $\tilde{\varphi} = 0$, will delays equally for one wavelength in comparison with the light falling on the other sample with a phase $\tilde{\varphi} = 2\pi$. For example, in the case of the reflective hologram for a sample with a phase $\tilde{\varphi} = 2\pi$ at wavelength λ =630 nm, the depth of a maximal relief modulation will be equal λ/2 i.e. 315 nm. If this phase matching has been achieved, all light falling on the kinoform will participate in formation a unique (real or imaginary) image of the object. Otherwise, the kinoform is similar to an axial hologram, where real and virtual images are partially superposed; some light is diffracted into the zeroth order, creating a bright spot in the center of the image.

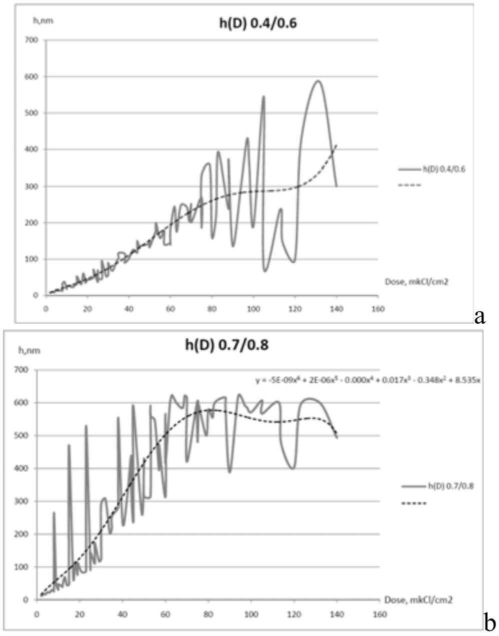

Fig. 4. Pulse response characteristics for different combinations of pixel's size and off-duty factor (peaks appeared due to Proximity Effect). a) 0.4/0.6 µm b) 0.7/0.8 µm.

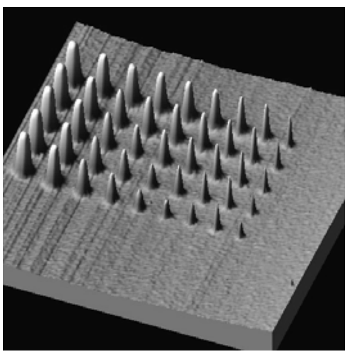

Fig. 5. Test structure recorded with optimized parameters.

We preliminary recorded a series of tests (Fig. 4) to find a pulse response characteristic of the recording system (Fig. 4) to choose optimum parameters of kinoform registration. Then test structure was recorded again with optimized parameters (Fig. 5) to find work range of exposure doses.

4. Results

As a result of using described algorithms, test samples of the CCGH and kinoform (with various values of the parameter *c* – less, more or equal to unit) have been recorded. Photos of the phase relief made using AFM (Fig. 6) and restored images (Fig.7) are shown below.

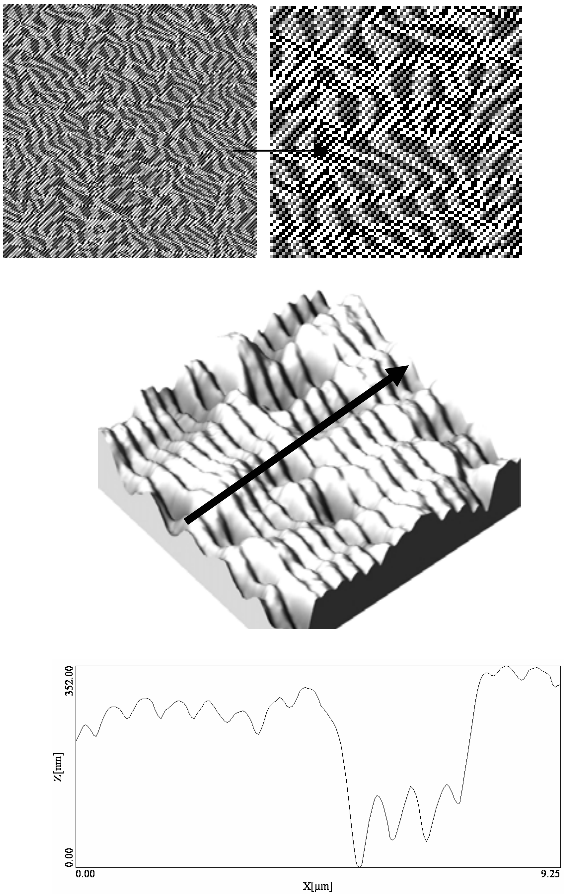

Fig. 6. Calculated IFD, surface of recorded CGH (received with AFM) and its profile.

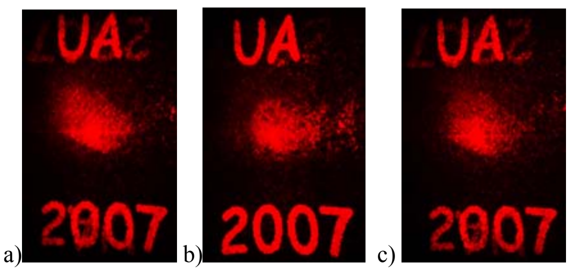

Fig. 7. Restored images $c = 0.7$ (a), 1 (b), 1.3 (c).

5. Summary

A problem of twin-image elimination in 2-D digital holograms is considered in this paper. Two approaches to solve this problem are proposed: using the quadrature holograms and kinoform methods. The offered method of a combined hologram has some advantages (quality of image is not strongly depends from the relief depth, because only horizontal positioning is used) and disadvantages (stable observing geometry required, spatial resolution as a rule is less than the 'vertical' one). But in general, both of these methods allows (without essential technology complexity of synthesis and record) receiving required result that is shown in photos of the images restored by recorded samples. From the practical viewpoint, it will allow to increase the resistance of the hologram to falsification (synthesis algorithm changes, demands to technological process and materials are increased), to increase intensity of the image restored (owing to energy re-distribution), to enlarge the possibilities for design solutions and to receive some extra features like easy relief-depth laser control and restoring different images for sources with various wavelengths.

Second meaning of this work is a preparation for work under more modern type of hologram - 3D computer-generated color hologram. Some preliminary job that was done already (as software for modelling or "dose - relief" data) will be used for further work. 3D color CGH is a powerful competitor to conventional rainbow optical hologram. Only few companies at the holography market can offer it. Besides, very interest result is expected in combination of optical (analogue) and E-Beam (digital) 3D holograms to have a bright (which is optical's advantage) and highly protected (due to possibilities of digital recording) security hologram type.

References

1. Lee W.H. Sampled Fourier-transform hologram generated by computer // *Appl. Opt.*, **9** (3), P.639-643 (1970).
2. Lee W.H. Computer-generated holograms: techniques and application. Ed. E. Wolf. N.Y.: Progr. Opt., Vol. 16, P.119-132 (1978).
3. Jaroslavski L.P., Merzlyakov N.S. Comparison of the methods of recording the computer-generated holograms - In Abstr. Symp. "Optika-80", 1980, P.86.
4. Chu D.C., Fienup J.F., Goodman J.W. Resent approaches to computer generated holograms // *Opt. Eng.*, **13** (3), P.189-195 (1974).
5. Brown B.R. Computer synthesis of holograms and spatial filters. – In: Application of holography, Ed. by Barrekette. N.Y.; L.1971, P. 215 – 227.
6. I.B. Lesem, P.M. Hirch, J.A. Jordan. The kinoform a new wave-front reconstruction device // *IBM Journal of Research and Develop.*, **13** (2), (1969).
7. L.M. Soroko, "Gilbert-optics", M., 1981, P.21-25.
8. D. Gabor, W. Goss, Interference microscope with total wave-front reconstruction" *J. Opt. Soc. Am.*, **56** (7), P.849 (1966)
9. Baltiyskiy Et Al., "Contemporary methods of digital holography", Problems of Coherent and Nonlinear Optics. S.-Pt., 2004, P.91-117.
10. E. Cuche, P. Marquet, Ch. Depeursinge, Spatial filtering for Zero-order and twin-image elimination in digital off-axis holography // *Applied Optics*, **39** (23), P. 4070-4075 (2000).
11. A.I. Fishman "The phase optical elements – kinoforms", Soros Review Journal (Sorosovskij Obrazovatel'nyj Zhurnal) No. 12, 1999, p. 76-83.
12. T.I. Kuznecova "About the phase problem in optics", *Uspekhi fisich. nauk*, **154** (4), p. 677-690 (1988).
13. B. Chhetri, S. Serikawa, S. Yang and etc. Binary phase difference diffuser: design and application in digital holography // *Opt. Eng.*, **41** (2), P.319-327 (2002).
14. Girnyk V., Kostyukevych S.., Kononov A., Borisov I. Multilevel computer-generated holograms for reconstructing 3D images in combined optical-digital security devices // *SPIE Proceedings* **4677,** P.255-266 (2002).
15. Girnyk V., Kononov A., Borisov I. Application of computer-generated rainbow holograms of 3D images in optical security // *SPIE Proceedings* **5005**, P. 338-344 (2003).
16. Xiangsu Z., Shou L., Xuechang R., Chuan L., Ying L. High quality 3D rainbow holograms with diffused illumination // *SPIE Proceedings,* **4924**, P.82-88 (2002).
17. Girnyk V., Kostyukevych S, Braginets E., Soroka A. 3D CGH registration on organic and non-organic resists: comparative analysis // *SPIE Proceedings,* **6136,** 61360P (2006).
18. Braginets E.V., Girnyk V.I., Kostyukevych S.A., Computer-generated holograms of 3D images in optical security devices // *SPIE Proceedings*, **5742**, P.33-40 (2005)
19. Girnyk V.I., Kostyukevych S.A., Shepeliavyi P.E., Kononov A.V., Borisov I.S., Multilevel computer-cenerated holograms for reconstructing 3-D images in combined optical digital security devices // *Semiconductor Physics, Quantum Electronics & Optoelectronics*, **5** (1), P.106-114 (2002).