ISSN 1560-8034, 1605-6582 (On-line), SPQEO, 2020. V. 23, N 1. P. 75-80.

Optics

Color temperature line: forward and inverse transformation

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Abstract. In the calculations, colorimetric calculations using different types of light sources used are the black body line and the family of isothermal lines but the coordinates of the light source are determined by a schedule or set of color coordinates. When using modern models of color type CAM02, CAM16 it is necessary to promptly receive the light source coordinate data knowing its color temperature. The paper proposes an analytical expression that binds the color temperature dependence and its location in the system *Yxy*. The presented results cover the Kelvin temperature range from 1080 up to 10,000 K. The algorithm has a certain conversion error, which is shown in the figures, but when evaluating its magnitude it was determined that these errors are beyond the threshold of human visibility. The paper presents results that can be used in the construction of modern adaptive to the source of illumination of photo and video transmission systems. The data describing the color body lines under different vision adaptation conditions are presented, which will allow considering these features in new systems, to make the perception of visual information maximum under different viewing conditions from ordinary to the extreme ones. The use of isothermal lines extends the functionality of the listed areas of application and simplifies working with them.

Keywords: color temperature, radiator strip, correlated color temperature, isothermal lines, CAM02, CAM16.

https://doi.org/10.15407/spqeo23.01.75
PACS 07.20.Dt, 42.66.Nc

Manuscript received 03.07.19; revised version received 26.11.19; accepted for publication 18.03.20; published online 23.03.20.

1. Introduction

The development of methods for describing light sources is based on the use of the data [1] where the authors propose to use the line color temperature and the correlated with it temperature (isothermal line). Isothermal lines in turn are represented in a limited amount, as well as an algorithm for finding intermediate values graphic-analytical consists in carrying out calculations. At the same time, finding the coordinates of the color temperature takes a long time.

To determine the source of illumination, chromaticity coordinates take a long time, and there is a need to identify it more and more often. The frequent use is caused by two factors. It should be attributed to the first measurement coordinate natural color of the light source, for example, bright sunlight, dusk, dawn, and so on. The second group should include a large variety of artificial light sources that have different spectral distribution and quality indicators.

Increasingly, there are systems that use dynamic periodic adjustment to the color temperature of the light source. The latter should include these [2, 3]. These systems are the basis for the construction of adaptive determination, management, and communication of color. That is why, it is necessary to develop the algorithm for dynamical determining the coordinate values of the light source.

2. Aims and objectives of the research

The purpose of the work is to develop a simplified algorithm for determining the coordinates of the light source of predetermined color temperature.

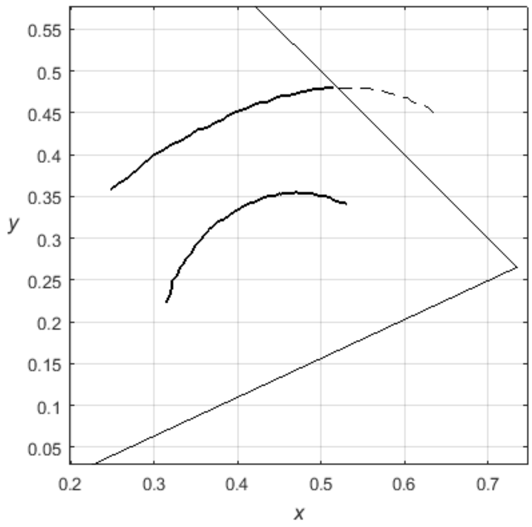

Fig. 1. Area describing the coordinates of the color temperature is constructed using the isothermal lines.

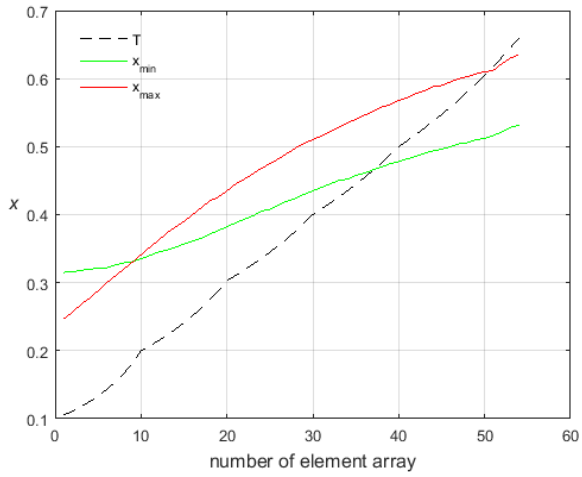

Fig. 2. Relationship between the coordinate values *x* and *T*.

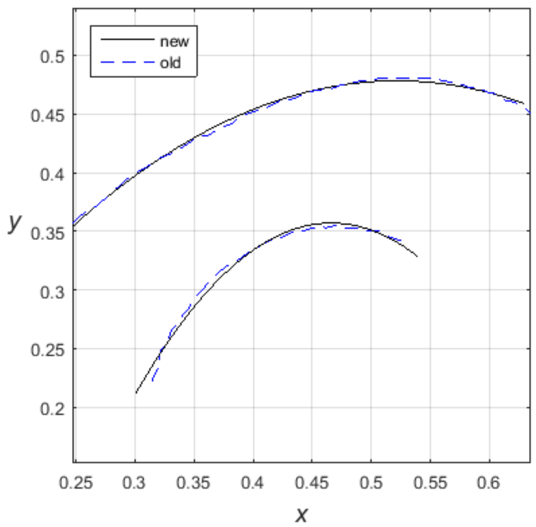

Fig. 3. Lines limiting the scope of the existence of the coordinates for light sources.

The objectives of the study include the following items:

- optimize the algorithm for determining the coordinates of the color temperature;
- find an analytical expression showing the relationship between color coordinates and color temperature; and
- perform studies of the proposed algorithm in an equidistant coordinate system.

3. Materials and methods

The line color temperature with isothermal lines is a region on the chromaticity diagram in coordinates *Yxy* (Fig. 1). A common line of color temperature is located approximately in the middle of isothermal lines. As can be seen from Fig. 1, the area on the chromaticity diagram can be represented in the form of lines, which are plotted accordingly to the use of extreme values of isothermal lines arrays.

As can also be seen from Fig. 1, part of the isothermal lines is outside the chromaticity diagram (shown by the dotted line), which suggests that some of the colors of sources values above 8000 K, which describes the isothermal lines, are outside the visible range and are likely to glow in such a manner that a source will be perceived by an observer as a saturated yellow.

If we consider the area of isothermal lines from the viewpoint of various industrial systems, the region of the transmitted color temperatures will be even less. For example, Fig. 2 shows that the region of color temperatures may transmit a high-definition system [4].

There are other systems with the ability to transmit more space chromaticity diagram. For these systems, to deliver color, region of origin will be greater. An example of a high-definition system is taken as a basis for the most common at this time – in control systems, automation, security, broadcasting, printing, *etc*.

Since the color coordinates can be varied, and for undistorted transmission it is necessary to consider them as shown in papers [5, 6], it is desirable to obtain an algorithm that describes the region shown in Fig. 1 as analytical formulas that can be implemented in existing and future color transmission systems and information about them.

3.1. Establishing the temperature relationship with the coordinate system xy

By studying the black body curve parameters, as well as the relationship with the color temperature with Fig. 1, it can be said that the value of *y* remains the same, regardless of whether we are talking about the *x* coordinate or *T* (color temperature). Therefore, the problem to establish the relationship between the color temperature and color coordinates is reduced to establishing the relationship coordinates *x* and *T*.

The relationship between the values of *x*, the lower limiting line of the upper, and *T* are shown in Fig. 2.

These values markedly vary greatly between themselves, what is why the interchangeability principle cannot be applied, when the color temperature value replaces the coordinate *x*. It should be added that the values of *x* vary in the repartition (0...1) and the value of *T* varies in the repartition (9500...1000 K), which complicates the implementation of the principle of interchangeability.

3.2. The region of existence of coordinates color temperature

To solve the problem of finding the relationship between the coordinates, a simulation was performed, where the initial data for the *x* coordinates and *T* are taken. Modeling is a determination of the functional dependence for describing the lines shown in Fig. 2. In order to achieve this goal, it is proposed to use the regression analysis of the first order, which is used in experimental studies to determine output depending on the current value.

The coordinate value of *T* should be brought into the form shown in Fig. 2. It should be a normalized coordinate value *T* with respect to the maximum value, hereinafter the normalized value of *T* will be represented as a value *t*, wherein $t = \frac{10000}{T}$ and $T \in \left(\overline{10000, 1515}\right)$ K (Kelvin temperatures).

After performing this research, the analytical expression was obtained (1), (2), which describes the line. The analytic expressions of the coordinates x_1, y_1 describe a lower boundary and x_2, y_2 – upper boundary. To determine the coefficient, it is used MATLAB simulation environment:

$$y_1 = -5.2997 \cdot x_1^2 + 4.9406 \cdot x_1 - 0.7944 , \quad (1)$$

$$y_2 = -1.6472 \cdot x_2^2 + 1.7185 \cdot x_2 + 0.0300 . \quad (2)$$

Solving the problem of analytical description of the lines shown in Fig. 2, we obtain a polynomial with two terms that describe the bottom boundary $x_{\min}$, the top $x_{\max}$, and *t*. The values of the parameter describe the functional dependence of the *t* different angles of inclination from the constant component values of dependences $x_{\min}$ and $x_{\max}$. The angular coordinates differ by (3)

$$\Delta\alpha = \mathrm{tg}\frac{\{x_{\min}(1),\ x_{\max}(1)\} - t(1)}{1 + \{x_{\min}(1),\ x_{\max}(1)\} \cdot t(1)}$$
$$\text{when } \angle(\{x_{\min}(1),\ x_{\max}(1)\}, t(1)) \neq 90°,$$

$$\Delta\alpha = \mathrm{ctg}\frac{\{x_{\min}(1),\ x_{\max}(1)\} - t(1)}{1 + \{x_{\min}(1),\ x_{\max}(1)\} \cdot t(1)} \quad (3)$$
$$\text{when } \angle(\{x_{\min}(1),\ x_{\max}(1)\}, t(1)) = 90°,$$

where $x(1)(1)(1)_{\max/\min}$ – the number of values of a polynomial describing the functional dependence.

The relationship between the values of the coordinates *x* and *T* can be expressed using the analytic expressions (4) and (5):

$$x_1 = -0.7588 \cdot t^2 + 0.2100 \cdot t + 0.1291, \quad (4)$$

$$x_2 = -0.0456 \cdot t^2 + 0.4613 \cdot t + 0.2549 . \quad (5)$$

The use of the expressions (1), (2) and (3), (4) allows to obtain analytical expressions describing the existence region of the color temperature coordinate. The values obtained after modeling as well as the experimental ones [1] are shown in Fig. 3.

3.3. Equation of Planck temperature line

Taking as an example the definition of the boundary lines for the existence region of the chromaticity coordinates of light sources expressions were determined as dependent on location x_T and setting *T*:

$$x_T = -0.4034 \cdot t^2 + 0.8562 \cdot t + 0.1925 . \quad (6)$$

The parameter y_T corresponds to the expression (7):

$$y_T = -2.6729 \cdot x_T^2 + 2.6615 \cdot x_T - 0.2490 \quad (7)$$

Some idea of the formula is given in [7] for the light sources of type D. In this work the formula for an arbitrary illumination source that regard (5) the analytical expression describing the horizontal coordinate is greatly simplified and easier to use for an engineer.

3.4. Isotropic line (correlated temperature)

Using the expressions (1)–(7), one can get an array coordinate values *x*, *y* for values *T* that are represented, for example, in [7]. Look of isothermal lines is shown in Fig. 5.

The unevenness of the lines in the figure can be seen due to the fact that the color temperature is not evenly distributed along the line of the black body.

The simulation results are from the experimental error of the data presented in the literature reference; the error is less than 0.05 for each coordinate axes of the space *x*, *y*.

3.5. Forward model. Determination of chromaticity coordinates in a known color temperature

If the color temperature (K) is determined, the coordinates *x* can be applied in the formula (6) to determine if coordinates *y* can be applied in the formula (7).

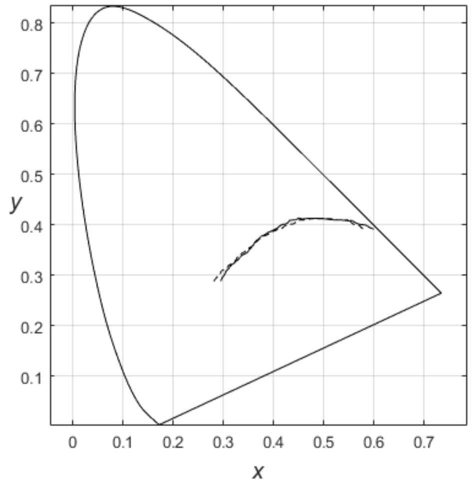

Fig. 4. Results of the black body line simulation (solid line – the resulting functional dependence, dashed – reference data).

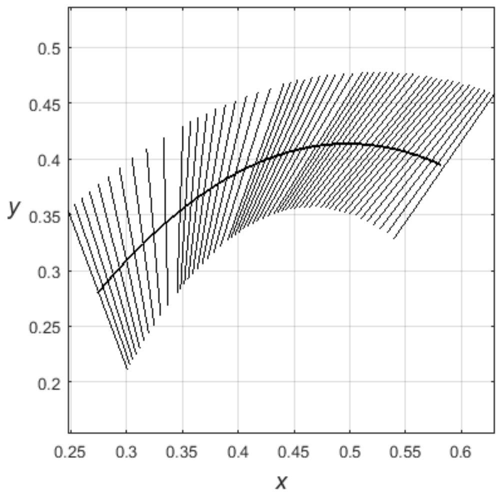

Fig. 5. Isothermal line and line of the black body obtained using the analytical expressions (1)–(7).

In case when the search coordinate values belonging to isothermal lines determine coordinates *x*, *y*, the lower boundary line and the top, two points of the segment obtained isothermal line will meet the predetermined color temperature.

3.6. The inverse model. Determination of the color temperature by using the chromaticity coordinates of the known

For the case when only one coordinate set color temperature, for example y_T, then it is necessary to determine the coordinates x_T. The relationship between the coordinates to the black body line is represented by the expression (8). Since the functional relationship between the coordinates of the extremum is therefore in the expression (8), there is a condition:

$$x_T = \sqrt{0.3741 y_T - 0.1546} + 0.4979 \quad \text{for} \quad y_T \geq 0.4122\,,$$
$$x_T = \sqrt{-0.3741 y_T + 0.1546} + 0.4979 \quad \text{for} \quad y_T < 0.4122\,. \quad (8)$$

For the obtained coordinate values or for the existing definition of color temperature, it is performed using the coordinates x_T:

$$T = \frac{1000}{\sqrt{-2.4789 x_T + 1.6033 + 1.0612}}\,. \quad (9)$$

3.7. Using the equation in the models of color

Several models, such as Lab and Luv, allow the description of color characteristics specified in the coordinates systems. The value of these coordinates is fair for one particular light source that is adapted on the basis of which the construction of these systems is realized. If one changes the source of illumination, there is a question about reliability, however.

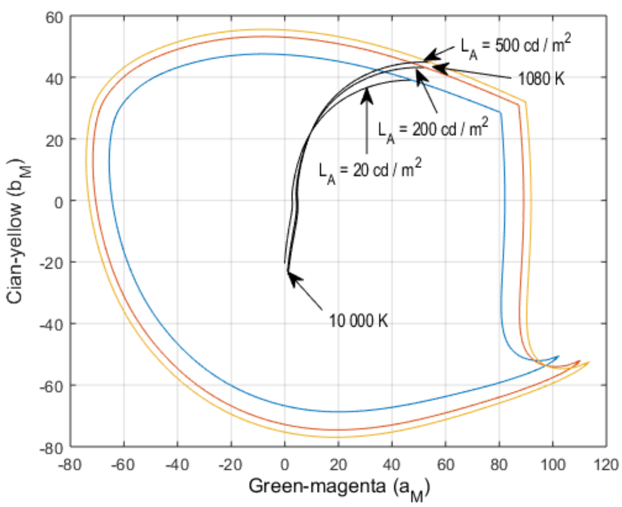

Fig. 6. Changes in the coordinates of the black body line, depending on the adapting luminance (L_A).

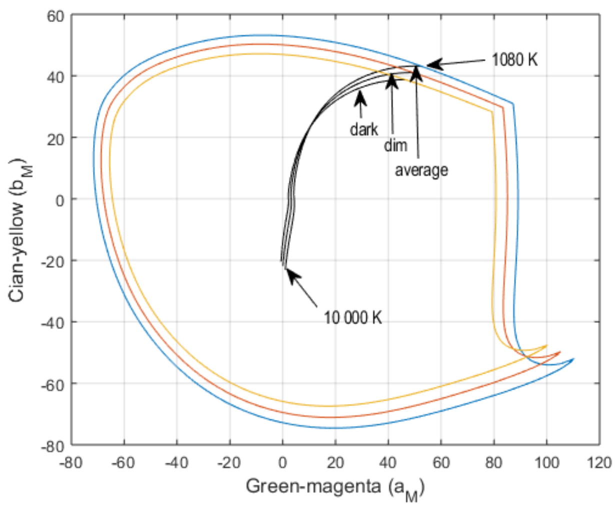

Fig. 7. Changes in the coordinates of the black body line, depending on the conditions of observation.

The mentioned alternative replacement systems are models of color CIECAM02, CAM02 and CAM16. In this latest model there are fewer inaccuracies that have taken place in previous versions. This model takes into account a number of adaptive properties of color, as well as the properties of color depending on the light source. The coordinates of the light source and the absolute values are given in the form of arrays of values *XYZ*.

```
T = [2000:10000];
t = 1000./T;
x = –0.4034.*t.^2+0.8562.*t+0.1925;
y = –2.6729.*x.^2+2.6615.*x–0.2490;
z = 1–x–y;
Y = 100;
X = x./y.*Y;
Z = z./y.*Y;
```

To use in the color perception model [2, 3], the black body line in accord with (6), (7) should be used for example in the following MATLAB script environment [8].

Data to rapidly produce both direct and inverse transformation can be applied in dynamic systems, such as the adaptive ones, if considering the influence of the light source.

4. Discussion

Some result of adapting is present in [9-13], but these do not use the color temperature line. The result is a black body line in a system of color coordinates of the model and the same values for different adaptation conditions. The data shown in Fig. 6 show how to change coordinates to the black body line dependences by adapting the brightness. Fig. 7 shows the change in coordinates depending on observation conditions – average, dim, dark. These results can be used to construct new and modernize image and video system [14-17]. Adapting video systems are future system and will be used in special and usual applications.

Being based on these figures, it can be concluded that the proposed algorithm directly inversely transforms the color temperature and color coordinate allowing to dynamically change the value of the coordinates of white color source in models such as CAM16 and others similar to it.

5. Conclusion

The presented analytical expressions (1)–(5) to determine the existence region of color coordinates and (6), (7) for determining the coordinates x_T, y_T of the black body line greatly simplify the respective calculations and provide an alternative tool to replace the graphic-analytical method.

The expressions allow the available color coordinates to define the color temperature, and *vice versa*, having just the color temperature value to determine the coordinates in the coordinate system luminescence x, y.

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