Optics

Optical soliton perturbation and conservation law with Kudryashov's refractive index having quadrupled power-law and dual form of generalized nonlocal nonlinearity

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Abstract. This paper is devoted to optical solitons for Kudryashov's law of nonlinear refractive index, which stem from quadrupled-power law and dual form of nonlocal nonlinearity. The conservation law has been also exhibited to paint a complete picture of the model.

Keywords: solitons, Kudryashov's law, perturbation, conservation law.

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1. Introduction

Optical soliton dynamics in metamaterials, couplers as well as PCF and hollow-core optical fibers have left a lasting impression thus far [1–20]. A wide spectrum of analytical results have been reported in several books and journals. These results deliver a variety of novel phenomena that includes performance enhancement all the way to technological novelty. There are still various aspects addressed in the context of fiber and nonlinear optics. Some of these newly developed concepts are cubic–quartic optical solitons, when chromatic dispersion (CD) maintains a low count, its extension to highly dispersive optical solitons both in polarization–preserving and birefringent fibers. New generalized forms of self–phase modulation (SPM) have also been proposed and these are attributed to Nikolay Kudryashov [5–13]. The current paper deals with one such form of SPM that was proposed by Kudryashov where a perturbed version of the governing nonlinear Schrödinger's equation (NLSE) is being studied. The perturbation terms are considered with maximum intensity. SPM comprises of four power–law nonlinear effects and a dual combination of nonlocal nonlinear effects. The model of this paper has been integrated to recover soliton solutions and yields a conserved quantity, which is also exhibited. The details have been given out after a quick intro to the perturbed NLSE.

1.1. Governing model

The NLSE with dual form of generalized nonlocal nonlinearity and perturbation terms is:

$$iq_t + aq_{xx} + \left(b_1|q|^n + b_2|q|^{2n} + b_3|q|^{3n} + b_4|q|^{4n}\right)q + \left\{c_1\left(|q|^n\right)_{xx} + c_2\left(|q|^{2n}\right)_{xx}\right\}q = i\left[\lambda\left(|q|^{2m}q\right)_x + \theta\left(|q|^{2m}\right)_x q + \mu|q|^{2m}q_x\right], \quad (1)$$

where x represents the non-dimensional distance, while t represents time in dimensionless form. $q(x, t)$ is the

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complex-valued dependent variable, and it represents the soliton profile. a – coefficient of chromatic dispersion (CD) and $i=\sqrt{-1}$. The constants b_j for $1\le j\le 4$ are the coefficients of nonlinearity effects, while the constants c_l for $l=1,2$ are the coefficients of dual form of generalized nonlocal nonlinearity. n is the power nonlinearity parameter, λ – coefficient of self-steepening for short pulses, θ – coefficient of higher-order dispersion, μ – coefficient of nonlinear dispersion, and m – full nonlinearity parameter.

2. Mathematical analysis

Optical solitons with the Kudryashov equation with dual form of generalized nonlocal nonlinearity and perturbation terms are described in this paper. Being aimed at it, we set the equation

$$q(x,t)=U(\xi)e^{i\varphi(x,t)}. \tag{2}$$

In Eq. (2), the amplitude component of the soliton is $U(\xi)$, where

$$\xi=\eta(x-vt) \tag{3}$$

and v and η are the velocity and width of the soliton respectively, while the phase component of the soliton is

$$\varphi(x,t)=-\kappa x+\omega t+\theta_0, \tag{4}$$

where κ,ω and θ_0 correspond to the frequency, wave number and phase constant of the soliton, respectively.

Inserting Eq. (2) into Eq. (1), the real and imaginary equations are

$$\begin{aligned}&\left(\omega+a\kappa^2\right)U-a\eta^2U''-\eta^2nc_1U^nU''+\\&+\eta^2c_1n(1-n)(U')^2U^{n-1}-2\eta^2nc_2U^{2n}U''+\\&+2\eta^2c_2n(1-2n)(U')^2U^{2n-1}-b_1U^{1+n}-\\&-b_2U^{1+2n}-b_3U^{1+3n}-b_4U^{1+4n}+\kappa(\lambda+\mu)U^{1+2m}=0\end{aligned} \tag{5}$$

and

$$v+2a\kappa+(\lambda+2\lambda m+\mu+2\theta m)P^{2m}=0, \tag{6}$$

respectively. From Eq. (6), we get the constraint conditions

$$v=-2a\kappa, \tag{7}$$

$$\lambda+2\lambda m+\mu+2\theta m=0, \tag{8}$$

while Eq. (5) is modified to

$$\begin{aligned}&n^2\left(\omega+a\kappa^2\right)Q^2-a\eta^2\left((1-n)\left(Q'\right)^2+nQQ''\right)-\\&-\eta^2n^2c_1Q^2Q''-2\eta^2n^2c_2Q^2\left(Q'\right)^2-\\&-2\eta^2n^2c_2Q^3Q''-b_1n^2Q^3-b_2n^2Q^4-\\&-b_3n^2Q^5-b_4n^2Q^6+n^2\kappa(\lambda+\mu)Q^{\frac{2m}{n}+2}=0\end{aligned} \tag{9}$$

by using $U=Q^{\frac{1}{n}}$. Eq. (8) gives the constraint relation between the perturbation terms, while the velocity of the optical solitons is emerged from the equation (7). The profile of the optical solitons is imparted after integrating the equation (9).

2.1. Trial equation scheme

The solution of Eq. (9) is given as

$$\left(Q'(\xi)\right)^2=\sum_{i=0}^{N}A_iQ^i(\xi), \tag{10}$$

where N is the balance number, A_i for $(0\le i\le N)$ are constants and $Q(\xi)$ is an unknown function to be determined later.

According to the balancing principle, the solution of Eq. (9) is

$$\left(Q'(\xi)\right)^2=A_0+A_1Q(\xi)+A_2Q^2(\xi)+A_3Q^3(\xi)+A_4Q^4(\xi). \tag{11}$$

Substituting Eq. (11) into Eq. (9), one gets the results

$$\begin{aligned}&m=2n,\ A_0=0,\ A_1=0,\\&A_2=\frac{n^2\left(a\kappa^2+\omega\right)}{a\eta^2},\\&A_3=-\frac{\kappa\lambda c_1+\kappa\mu c_1+3b_3c_2-b_4c_1}{15c_2{}^2\eta^2},\end{aligned} \tag{12}$$

$$\begin{aligned}&A_4=\frac{\lambda\kappa+\mu\kappa-b_4}{6\eta^2c_2},\\&b_1=\frac{-30n^4c_1c_2{}^2\left(a\kappa^2+\omega\right)+3a^2b_3c_2(n+2)+a^2c_1(n+2)(\kappa\lambda+\kappa\mu-b_4)}{30ac_2{}^2n^2},\end{aligned} \tag{13}$$

$$b_2=-\frac{120n^4c_2{}^3\left(a\kappa^2+\omega\right)-9an^2b_3c_1c_2-3an^2c_1{}^2(\kappa\lambda+\kappa\mu-b_4)+5a^2c_2(n+1)(\kappa\lambda+\kappa\mu-b_4)}{30ac_2{}^2n^2}. \tag{14}$$

Plugging Eqs. (12)-(14) into Eq. (11), we get

$$\pm(\xi-\xi_0)=\int\frac{dQ}{\sqrt{\frac{n^2(a\kappa^2+\omega)}{a\eta^2}Q^2-\frac{\kappa\lambda c_1+\kappa\mu c_1+3b_3c_2-b_4c_1}{15c_2{}^2\eta^2}Q^3+\frac{\lambda\kappa+\mu\kappa-b_4}{6\eta^2c_2}Q^4}}. \tag{15}$$

Case 1:
If we set $\lambda\kappa+\mu\kappa-b_4=0$ in Eq. (15) and integrating with respect to Q, bright soliton is

$$q(x,t)=\left(\begin{array}{l}\pm n\sqrt{\frac{5c_2(a\kappa^2+\omega)}{ab_3}}\times\\ \times\operatorname{sech}\left[\frac{n}{2}\sqrt{\frac{a\kappa^2+\omega}{a}}(x+2a\kappa)\right]\end{array}\right)^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta_0)} \tag{16}$$

and singular soliton is

$$q(x,t)=\left(\begin{array}{l}\pm n\sqrt{-\frac{5c_2(a\kappa^2+\omega)}{ab_3}}\times\\ \times\operatorname{csch}\left[\frac{n}{2}\sqrt{\frac{a\kappa^2+\omega}{a}}(x+2a\kappa)\right]\end{array}\right)^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta_0)}. \tag{17}$$

The bright soliton is valid for

$$a(a\kappa^2+\omega)>0, \tag{18}$$

$$ab_3c_2(a\kappa^2+\omega)>0, \tag{19}$$

while the singular soliton is valid for the constraint (18) along with

$$ab_3c_2(a\kappa^2+\omega)<0. \tag{20}$$

Case 2:

If we set $a\kappa^2+\omega=\frac{a(\kappa\lambda c_1+\kappa\mu c_1+3b_3c_2-b_4c_1)^2}{150n^2c_2^3(\lambda\kappa+\mu\kappa-b_4)}$ in

Eq. (15) and integrating with respect to Q, dark soliton is

$$q(x,t)=\left(\begin{array}{l}\frac{\kappa\lambda c_1+\kappa\mu c_1+3b_3c_2-b_4c_1}{10c_2(\lambda\kappa+\mu\kappa-b_4)}\pm\frac{\kappa\lambda c_1+\kappa\mu c_1+3b_3c_2-b_4c_1}{10c_2(\lambda\kappa+\mu\kappa-b_4)}\\ \times\tanh\left[\frac{\kappa\lambda c_1+\kappa\mu c_1+3b_3c_2-b_4c_1}{10c_2}\sqrt{\frac{1}{6c_2(\lambda\kappa+\mu\kappa-b_4)}}(x+2a\kappa)\right]\end{array}\right)^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta_0)} \tag{21}$$

and singular soliton is

$$q(x,t)=\left(\begin{array}{l}\frac{\kappa\lambda c_1+\kappa\mu c_1+3b_3c_2-b_4c_1}{10c_2(\lambda\kappa+\mu\kappa-b_4)}\pm\frac{\kappa\lambda c_1+\kappa\mu c_1+3b_3c_2-b_4c_1}{10c_2(\lambda\kappa+\mu\kappa-b_4)}\\ \times\coth\left[\frac{\kappa\lambda c_1+\kappa\mu c_1+3b_3c_2-b_4c_1}{10c_2}\sqrt{\frac{1}{6c_2(\lambda\kappa+\mu\kappa-b_4)}}(x+2a\kappa)\right]\end{array}\right)^{\frac{1}{n}}e^{i(-\kappa x+\omega t+\theta_0)}. \tag{22}$$

These solitons are valid for

$$c_2(\lambda\kappa+\mu\kappa-b_4)>0. \tag{23}$$

2.2. Sine–Gordon equation approach

The solution of Eq. (9) is given as

$$Q(\xi)=\sum_{i=1}^{N}\cos^{i-1}(V(\xi))[B_i\sin(V(\xi))+A_i\cos(V(\xi))]+A_0, \tag{24}$$

where N is the balance number, A_i and B_i for $(0\le i\le N)$ are constants and the function $V(\xi)$ satisfies

$$V'(\xi)=\sin(V(\xi)). \tag{25}$$

The solutions of Eq.(25) are:

$$\sin(V(\xi))=\operatorname{sech}(\xi)\ \text{or}\ \sin(V(\xi))=\mathrm{icsh}(\xi),$$

$$\cos(V(\xi))=\tanh(\xi)\ \text{or}\ \cos(V(\xi))=\coth(\xi). \tag{26}$$

According to the balancing principle, the solution of Eq. (9) is

$$Q(\xi)=B_1\sin(V(\xi))+A_1\cos(V(\xi))+A_0. \tag{27}$$

Substituting Eq. (27) along with Eq. (25) into Eq. (9), one gets the results.

Case 1:

$$m = 2n,\ \eta = \pm\sqrt{\frac{n^2\left(a\kappa^2+\omega\right)}{4a}},\ A_0 = \pm\sqrt{\frac{3n^2c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}},\ A_1 = \pm\sqrt{\frac{3n^2c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}}, \tag{28}$$

$$B_1 = 0,\ b_1 = -\frac{n^2c_1\left(\omega+a\kappa^2\right)}{a} \pm \frac{a(n+2)(\kappa\lambda+\kappa\mu-b_4)}{3nc_2}\sqrt{\frac{3c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}}, \tag{29}$$

$$b_2 = -\frac{a(n+1)(\kappa\lambda+\kappa\mu-b_4)}{6c_2n^2} - \frac{4n^2c_2\left(a\kappa^2+\omega\right)}{a} \pm \frac{c_1n(\kappa\lambda+\kappa\mu-b_4)}{c_2}\sqrt{\frac{3c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}}, \tag{30}$$

$$b_3 = -\frac{c_1(\kappa\lambda+\kappa\mu-b_4)}{3c_2} \pm \frac{10nc_2(\kappa\lambda+\kappa\mu-b_4)}{3c_2}\sqrt{\frac{3c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}}. \tag{31}$$

Inserting Eqs (28)-(31) along with Eq. (26) into Eq. (27), dark soliton is

$$q(x,t) = \left(\pm n\sqrt{\frac{3c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}} \pm n\sqrt{\frac{3c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}}\tanh\left[\frac{n}{2}\sqrt{\frac{a\kappa^2+\omega}{a}}(x+2a\kappa)\right]\right)^{\frac{1}{n}} e^{i\left(-\kappa x+\omega t+\theta_0\right)} \tag{32}$$

and singular soliton is

$$q(x,t) = \left(\pm n\sqrt{\frac{3c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}} \pm n\sqrt{\frac{3c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}}\coth\left[\frac{n}{2}\sqrt{\frac{a\kappa^2+\omega}{a}}(x+2a\kappa)\right]\right)^{\frac{1}{n}} e^{i\left(-\kappa x+\omega t+\theta_0\right)}. \tag{33}$$

These solitons are valid for the constraint (18) along with

$$ac_2\left(a\kappa^2+\omega\right)(\kappa\lambda+\kappa\mu-b_4) > 0. \tag{34}$$

Case 2:

$$m = 2n,\ \eta = \pm\sqrt{\frac{n^2\left(a\kappa^2+\omega\right)}{a}},\ A_0 = 0,\ A_1 = 0,\ B_1 = \pm\sqrt{-\frac{6n^2c_2\left(a\kappa^2+\omega\right)}{a\left(\kappa\lambda+\kappa\mu-b_4\right)}}, \tag{35}$$

$$b_1 = -\frac{n^2\left(a\kappa^2+\omega\right)c_1}{a},\ b_3 = -\frac{c_1(\kappa\lambda+\kappa\mu-b_4)}{3c_2}, \tag{36}$$

$$b_2 = -\frac{24n^4c_2^{\,2}\left(a\kappa^2+\omega\right)+a^2(n+1)(\kappa\lambda+\kappa\mu-b_4)}{6c_2an^2}. \tag{37}$$

Plugging Eqs (35)-(37) along with Eq. (26) into Eq. (27), bright soliton is

$$q(x,t) = \left(\pm n\sqrt{-\frac{6c_2\left(a\kappa^2+\omega\right)}{a\left(\kappa\lambda+\kappa\mu-b_4\right)}}\,\mathrm{sech}\left[n\sqrt{\frac{a\kappa^2+\omega}{a}}(x+2a\kappa)\right]\right)^{\frac{1}{n}} e^{i\left(-\kappa x+\omega t+\theta_0\right)}. \tag{38}$$

and singular soliton is

$$q(x,t)=\left(\pm n\sqrt{\frac{6c_2\left(a\kappa^2+\omega\right)}{a\left(\kappa\lambda+\kappa\mu-b_4\right)}}\operatorname{csch}\left[n\sqrt{\frac{a\kappa^2+\omega}{a}}\left(x+2a\kappa\right)\right]\right)^{\frac{1}{n}}e^{i\left(-\kappa x+\omega t+\theta_0\right)}. \tag{39}$$

The singular soliton is valid for the constraints (18) and (34), while the bright soliton is valid for the constraint (18) along with

$$ac_2\left(a\kappa^2+\omega\right)\left(\kappa\lambda+\kappa\mu-b_4\right)<0. \tag{40}$$

Case 3:

$$m=2n,\ \ \eta=\pm\sqrt{\frac{n^2\left(a\kappa^2+\omega\right)}{a}},\ \ A_0=\pm\sqrt{\frac{3n^2c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}},\ \ A_1=\pm\sqrt{\frac{3n^2c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}}, \tag{41}$$

$$B_1=\pm\sqrt{-\frac{3n^2c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}},\ \ b_1=-\frac{n^2c_1\left(a\kappa^2+\omega\right)}{a}\pm\frac{(n+2)\left(a\kappa^2+\omega\right)}{2n}\sqrt{\frac{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}{3c_2\left(a\kappa^2+\omega\right)}}, \tag{42}$$

$$b_2=-\frac{4n^2c_2\left(a\kappa^2+\omega\right)}{a}-\frac{a(n+1)\left(\kappa\lambda+\kappa\mu-b_4\right)}{6c_2n^2}\pm\frac{nc_1\left(\kappa\lambda+\kappa\mu-b_4\right)}{c_2}\sqrt{\frac{3c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}}, \tag{43}$$

$$b_3=-\frac{c_1\left(\kappa\lambda+\kappa\mu-b_4\right)}{3c_2}\pm\frac{5nc_2\left(a\kappa^2+\omega\right)}{a}\sqrt{\frac{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}{3c_2\left(a\kappa^2+\omega\right)}}. \tag{44}$$

Plugging Eqs (41)-(44) along with Eq. (26) into Eq. (27), combo singular soliton is

$$q(x,t)=\left(\begin{array}{l}\pm n\sqrt{\dfrac{3c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}}\pm n\sqrt{\dfrac{3c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}}\coth\left[n\sqrt{\dfrac{a\kappa^2+\omega}{a}}\left(x+2a\kappa\right)\right]\\ \pm n\sqrt{\dfrac{3c_2\left(a\kappa^2+\omega\right)}{2a\left(\kappa\lambda+\kappa\mu-b_4\right)}}\operatorname{csch}\left[n\sqrt{\dfrac{a\kappa^2+\omega}{a}}\left(x+2a\kappa\right)\right]\end{array}\right)^{\frac{1}{n}}e^{i\left(-\kappa x+\omega t+\theta_0\right)}. \tag{45}$$

The soliton is valid for the constraints (18) and (34).

3. Conservation law

The governing model as given by (1) supports only one conservation law, namely, the power (P). The bright 1–soliton solution to (1) is given as:

$$q(x,t)=A\operatorname{sech}^{\frac{1}{n}}\left[B(x-vt)\right]e^{i\left(-\kappa x+\omega t+\theta_0\right)}, \tag{46}$$

where A is the amplitude of the soliton, while B – inverse width. Thus, by virtue of this bright 1–soliton structure, the power is:

$$P=\int_{-\infty}^{\infty}|q|^2dx=\frac{A^2}{B}\frac{\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}. \tag{47}$$

4. Conclusions

This paper serves as an extension of the results to the unperturbed model that has been reported earlier [16]. The Hamiltonian type perturbations have been included with maximum intensity, which led to formulation of NLSE with strong perturbation terms. Its soliton solutions are recovered along with a conservation law. Thus, a full forefront picture to the model has evolved in this paper. Further studies are definitely pending, and they are with regards to application of soliton perturbation theory to recover the adiabatic parameter dynamics, implementation of variational principle, semi–inverse variational principle, method of moments, collective variables, Adomian decomposition scheme followed by Laplace–Adomian decomposition scheme, and many more. The results of such research activities are on their way to be exhibited, in several journals, across the global sphere.

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Збурення оптичного солітону та закон збереження з показником заломлення по Кудряшову, що має степеневу залежність четвертого порядку та дуальну форму узагальненої нелокальної нелінійності

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Анотація. У цій роботі досліджено оптичні солітони, що описуються законом Кудряшова з нелінійним показником заломлення, який випливає із степеневої залежності четвертого порядку та дуальної форми нелокальної нелінійності. Продемонстровано також, як закон збереження дозволяє доповнити картину моделі.

Ключові слова: солітони, закон Кудряшова, збурення, закон збереження.