Optics

Dispersive optical soliton perturbation with multiplicative white noise having parabolic law of self-phase modulation

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Abstract. In this paper, we investigate dispersive optical solitons incorporating multiplicative white noise. Utilizing the *F*-expansion procedure, we derive various soliton solutions, including dark soliton solutions, singular soliton solutions, bright soliton solutions, straddled singular-singular soliton solutions, complexiton solutions, and straddled dark-bright soliton solutions. Moreover, we discuss the parametric restrictions necessary for the existence of these soliton solutions, providing a comprehensive analysis of the conditions under which these solutions are valid. Our findings contribute to the understanding of the dynamics of perturbed nonlinear wave equations in the presence of stochastic influences.

Keywords: solitons, white noise, *F*-expansion.

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1. Introduction

Study of nonlinear wave equations has garnered significant attention due to their wide range of applications in various fields such as fluid dynamics, plasma physics, optical fibers, and quantum mechanics. Among these, nonlinear Schrödinger equation is particularly notable for its ability to describe complex wave phenomena, including soliton dynamics [1–10]. Solitons are stable, localized wave packets that maintain their shape while propagating at constant velocity, making them crucial in understanding nonlinear wave behavior. In real-world scenarios, wave propagation is often influenced by stochastic perturbations. These random influences can significantly alter the dynamics described by deterministic models. Incorporating such effects into the modeling of nonlinear wave equations is essential for accurately capturing the behavior of physical systems. The perturbed cubic-quartic nonlinear Schrödinger equation, which includes multiplicative white noise, represents a more realistic model by accounting for these stochastic effects. To analyze the solutions of this perturbed equation, various mathematical methods can be employed. One effective approach is the *F*-expansion procedure, which facilitates derivation of exact soliton solutions. This method has been successfully applied to various nonlinear equations, yielding a spectrum of soliton solutions that enhance our understanding of the underlying dynamics. In this paper, we apply the *F*-expansion procedure to the perturbed cubic-quartic nonlinear Schrödinger equation with multiplicative white noise. Our goal is to obtain different types of soliton solutions, including dark soliton solutions, singular soliton solutions, bright soliton solutions, straddled singular-singular soliton solutions, complexiton solutions, and straddled dark-bright soliton solutions. Moreover, we explore the parametric restrictions necessary for these solutions to exist, providing a comprehensive analysis of the conditions that govern soliton formation and stability.

The structure of the paper is as follows: after the introduction, we outline the mathematical formulation of the perturbed cubic-quartic nonlinear Schrödinger equation. We then describe the *F*-expansion procedure and apply it to derive various soliton solutions. Following this, we discuss the parametric restrictions for these solutions. Finally, we conclude with a summary of our findings and potential directions for future research. This study aims to contribute to the field of nonlinear wave equations by offering new insights into the soliton solutions of the perturbed cubic-quartic nonlinear Schrödinger equation in the presence of stochastic influences. Our results not only enhance the theoretical understanding of soliton dynamics but also have practical implications for systems where noise plays a crucial role.

1.1. Governing model

The dimensionless structure of the model equation that describes dispersive optical solitons incorporating multiplicative white noise can be expressed as follows:

$$iq_t + a_1 q_{xx} + ia_2 q_{xxx} + a_3 q_{xxxx} + (b_1|q|^2 + b_2|q|^4)q + \sigma q\frac{dW(t)}{dt} = i[\alpha(|q|^2 q)_x + \beta|q|^2 q_x + \gamma(|q|^2)_x q]. \quad (1)$$

The equation involves a complex-valued function $q(x,t)$, where x and t represent the spatial and temporal coordinates, respectively. The imaginary unit is given by i. The equation comprises several terms, including the evolution term, which governs the temporal dynamics of the wave profile. The other terms are related to dispersion phenomena, such as chromatic dispersion, cubic dispersion and quartic dispersion, represented by the coefficients a_1, a_2, and a_3, respectively. Additionally, the equation includes coefficients representing self-phase modulation, denoted by b_1 and b_2. There are also perturbation terms represented by the parameters α, β, and γ, which account for the effects of external factors on wave propagation. Here, σ signifies the coefficient of noise strength and $W(t)$ corresponds to the standard Wiener process, so that $\frac{dW(t)}{dt}$ expresses the white noise.

2. *F*-expansion procedure

We take into account the model equation:

$$G(q, q_x, q_t, q_{xt}, q_{xx}, \ldots) = 0, \quad (2)$$

and the constraints

$$q(x,t) = U(\xi),\ \xi = \mu(x - vt), \quad (3)$$

where ξ and μ take on the roles of the wave variable and wave width, respectively, and v signifies the wave velocity. It follows that Eq. (2) becomes

$$P(U, -\mu v U', \mu U', \mu^2 U'', \ldots) = 0. \quad (4)$$

Step–1: In the presence of (4), the simplified model confirms the solution structure

$$U(\xi) = \sum_{i=0}^{N} B_i F^i(\xi), \quad (5)$$

applying the ancillary equation

$$F'(\xi) = \sqrt{PF^4(\xi) + QF^2(\xi) + R}. \quad (6)$$

Thus, the soliton wave profiles derived from (6) are presented as follows:

$$\begin{cases} F(\xi) = \mathrm{sn}(\xi) = \tanh(\xi), \quad P = m^2, \\ Q = -(1+m^2), \quad R = 1, \quad m \to 1^-, \\ F(\xi) = \mathrm{ns}(\xi) = \coth(\xi), \quad P = 1, \\ Q = -(1+m^2), \quad R = m^2, \quad m \to 1^-, \\ F(\xi) = \mathrm{cn}(\xi) = sech\,(\xi), \quad P = -m^2, \\ Q = 2m^2 - 1, \quad R = 1 - m^2, \quad m \to 1^-, \\ F(\xi) = \mathrm{ds}(\xi) = csch\,(\xi), \quad P = 1, \\ Q = 2m^2 - 1, \ R = -m^2(1-m^2), m \to 1^-, \\ F(\xi) = \mathrm{ns}(\xi) \pm \mathrm{ds}(\xi) = \coth(\xi) \pm csch\,(\xi), \\ P = \frac{1}{4}, \quad Q = \frac{m^2-2}{2}, \quad R = \frac{m^2}{4}, \quad m \to 1^- \\ F(\xi) = \mathrm{sn}(\xi) \pm \mathrm{icn}(\xi) = \tanh(\xi) \pm \mathrm{i}\, sech\,(\xi), \\ P = \frac{m^2}{4}, \ Q = \frac{m^2-2}{2}, \ R = \frac{m^2}{4}, \quad m \to 1^-, \\ F(\xi) = \frac{\mathrm{sn}(\xi)}{1 \pm \mathrm{dn}(\xi)} = \frac{\tanh(\xi)}{1 \pm sech}, \\ P = \frac{m^2}{4}, \ Q = \frac{m^2-2}{2}, \ R = \frac{m^2}{4}, m \to 1^-, \end{cases} \quad (7)$$

where the Jacobi elliptic functions (JEFs) sn(ξ), ns(ξ), cn(ξ), ds(ξ) and dn(ξ) are associated with a modulus, $0 < m < 1$. Furthermore, the constants B_i (with i ranging from 0 to N) are a product of the balancing approach outlined in (4).

Step–2: Combining (5) and (6) within (4), we establish a system of equations that leads to determination of the unknown constants in (4) through (7).

3. Optical solitons

In this section, the integration method is employed to acquire optical solitons in conjunction with the model. The optical soliton is characterized by the assumed profile:

$$q(x,t) = U(\xi)e^{i\vartheta(x,t)}. \quad (8)$$

Here, $U(\xi)$ is the soliton amplitude component, while the wave variable is given by

$$\xi = x - vt, \quad (9)$$

where μ is the wave width and v is the velocity. Also, the soliton phase component is given by

$$\vartheta(x,t) = -\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0. \quad (10)$$

Here, κ denotes the soliton frequency, ω denotes the wave number, and θ_0 is the phase constant, respectively.

Substituting (2) into (1) and separating the result into real and imaginary components, one obtains:

$$a_3U^{(iv)} + (a_1 + 3\kappa\, a_2 - 6\, a_3\kappa^2)U'' + (\sigma^2 - \omega - \kappa^2 a_1 + a_3\kappa^4 - \kappa^3 a_2)U + (b_1 - \alpha\,\kappa - \beta\,\kappa)U^3 + b_2U^5 = 0, \quad (11)$$

and

$$(a_2 - 4\, a_3\kappa)U''' + (4\, a_3\kappa^3 - 2\kappa a_1 - v - 3\kappa^2 a_2)U' - -(2\gamma + \beta + 3\alpha)U^2U' = 0. \quad (12)$$

From (12), we get the following constraint relations:

$$a_2 = 4\, a_3\kappa,$$

and

$$3\alpha + \beta + 2\gamma = 0. \quad (13)$$

After implementing these changes, the soliton velocity derived from (12) can be written as follows:

$$v = -2\kappa a_1 - 8\kappa^3 a_3, \quad (14)$$

and the governing equation (11) is transformed to

$$a_3U^{(iv)} + (a_1 + 6\, a_3\kappa^2)U'' + (\sigma^2 - \omega - \kappa^2 a_1 - 3\kappa^4 a_3)U + (b_1 - \alpha\,\kappa - \beta\,\kappa)U^3 + b_2U^5 = 0. \quad (15)$$

The balance of the terms U^5 and $U^{(4)}$ in (15) results in $N = 1$. In this integration process, the solution structure (5) is presented in a simplified form as

$$U(\xi) = B_0 + B_1F(\xi). \quad (16)$$

Combination of (16) with (6) into (15) leaves us with the following equations:

$$\begin{cases} 5\, B_0{B_1}^4 b_2 = 0, \\ \sigma^2 B_0 - \omega\, B_0 - \alpha\,\kappa\, {B_0}^3 - \beta\,\kappa\, {B_0}^3 + {B_0}^3 b_1 + {B_0}^5 b_2 + \kappa^4 B_0 a_3 - \kappa^3 B_0 a_2 - \kappa^2 B_0 a_1 = 0, \\ 12\, PRB_1a_3 - 6\, Q\kappa^2 B_1 a_3 + 3\, Q\kappa\, B_1 a_2 - 3\,\alpha\,\kappa\, {B_0}^2 B_1 - 3\,\beta\,\kappa\, {B_0}^2 B_1 + Q^2 B_1 a_3 + QB_1a_1 + 3\, {B_0}^2 B_1 b_1 + 5\, {B_0}^4 B_1 b_2 + \kappa^4 B_1 a_3 - \kappa^3 B_1 a_2 - \kappa^2 B_1 a_1 + \sigma^2 B_1 - \omega\, B_1 = 0, \\ {B_1}^5 b_2 + 24\, P^2 B_1 a_3 = 0, \\ 10\, {B_0}^3 {B_1}^2 b_2 - 3\,\alpha\,\kappa\, B_0 {B_1}^2 - 3\,\beta\,\kappa\, B_0 {B_1}^2 + 3\, B_0 {B_1}^2 b_1 = 0, \\ 10\, {B_0}^2 {B_1}^3 b_2 - 12\, P\kappa^2 B_1 a_3 - \alpha\,\kappa\, {B_1}^3 - \beta\,\kappa\, {B_1}^3 + 20\, PQB_1a_3 + 6\, P\kappa\, B_1 a_2 + {B_1}^3 b_1 + 2\, PB_1a_1 = 0. \end{cases} \quad (17)$$

Upon solving these equations, one uncovers the outcomes:

$$\begin{cases} B_0 = 0, \quad B_1 = \pm\sqrt{-\dfrac{12\, P\kappa^2 a_3 - 20\, PQa_3 - 6\, P\kappa\, a_2 - 2\, Pa_1}{\alpha\,\kappa + \beta\,\kappa - b_1}}, \\ \omega = \kappa^4 a_3 - 6\, Q\kappa^2 a_3 - \kappa^3 a_2 + 12\, PRa_3 + Q^2 a_3 + 3\, Q\kappa\, a_2 - \kappa^2 a_1 + Qa_1 + \sigma^2, \\ b_2 = -\dfrac{6a_3(\alpha^2\kappa^2 + 2\,\alpha\,\beta\,\kappa^2 + \beta^2\kappa^2 - 2\,\alpha\,\kappa\, b_1 - 2\,\beta\,\kappa\, b_1 + b_1^2)}{36\,\kappa^4 a_3^2 - 120\, Q\kappa^2 a_3^2 - 36\,\kappa^3 a_2 a_3 + 100\, Q^2 a_3^2 + 60\, Q\kappa\, a_2 a_3 - 12\,\kappa^2 a_1 a_3 + 9\,\kappa^2 a_2^2 + 20\, Qa_1a_3 + 6\,\kappa\, a_1 a_2 + a_1^2} \end{cases} \quad (18)$$

Result 1:

Referring to (7), Eq. (18) evolves into

$$\begin{cases} B_0 = 0, \quad B_1 = \pm\sqrt{-\dfrac{12\,\kappa^2 a_3 - 6\,\kappa\, a_2 - 2\, a_1 + 40\, a_3}{\alpha\,\kappa + \beta\,\kappa - b_1}}, \\ \omega = \kappa^4 a_3 - \kappa^3 a_2 - \kappa^2 a_1 + 12\,\kappa^2 a_3 - 6\,\kappa\, a_2 + \sigma^2 - 2\, a_1 + 16\, a_3, \\ b_2 = -\dfrac{6a_3(\alpha^2\kappa^2 + 2\,\alpha\,\beta\,\kappa^2 + \beta^2\kappa^2 - 2\,\alpha\,\kappa\, b_1 - 2\,\beta\,\kappa\, b_1 + b_1^2)}{36\,\kappa^4 a_3^2 - 36\,\kappa^3 a_2 a_3 - 12\,\kappa^2 a_1 a_3 + 9\,\kappa^2 a_2^2 + 240\,\kappa^2 a_3^2 + 6\,\kappa\, a_1 a_2 - 120\,\kappa\, a_2 a_3 + a_1^2 - 40\, a_1 a_3 + 400\, a_3^2} \end{cases} \quad (19)$$

In conclusion, the dark and singular soliton solutions are formulated as

$$q(x,t) = \pm\sqrt{-\frac{12\,\kappa^2 a_3 - 6\,\kappa\, a_2 - 2\, a_1 + 40\, a_3}{\alpha\,\kappa + \beta\,\kappa - b_1}}\tanh(x + (2\kappa a_1 + 8\kappa^3 a_3)t)e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0)}, \quad (20)$$

and

$$q(x,t) = \pm\sqrt{-\frac{12\,\kappa^2 a_3 - 6\,\kappa\, a_2 - 2\, a_1 + 40\, a_3}{\alpha\,\kappa + \beta\,\kappa - b_1}}\coth(x + (2\kappa a_1 + 8\kappa^3 a_3)t)e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0)}. \quad (21)$$

The wave forms in Eqs. (20) and (21) are depicted by

$$(\alpha\,\kappa+\beta\,\kappa-b_1)(12\,\kappa^2a_3-6\,\kappa\,a_2-2\,a_1+40\,a_3)<0 \quad (22)$$

Result 2:

Utilizing (7), Eq. (18) transforms into

$$\begin{cases} B_0=0, \quad B_1=\pm\sqrt{\frac{12\,\kappa^2a_3-6\,\kappa\,a_2-2\,a_1-20\,a_3}{\alpha\,\kappa+\beta\,\kappa-b_1}}, \\ \omega=\kappa^4a_3-\kappa^3a_2-\kappa^2a_1-6\,\kappa^2a_3+3\,\kappa\,a_2+\sigma^2+a_1+a_3, \\ b_2=-\frac{6a_3(\alpha^2\kappa^2+2\,\alpha\,\beta\,\kappa^2+\beta^2\kappa^2-2\,\alpha\,\kappa\,b_1-2\,\beta\,\kappa\,b_1+{b_1}^2)}{36\,\kappa^4{a_3}^2-36\,\kappa^3a_2a_3-12\,\kappa^2a_1a_3+9\,\kappa^2{a_2}^2-120\,\kappa^2{a_3}^2+6\,\kappa\,a_1a_2+60\,\kappa\,a_2a_3+{a_1}^2+20\,a_1a_3+100\,{a_3}^2}. \end{cases} \quad (23)$$

Thus, the solution for the bright soliton takes the form of

$$q(x,t)=\pm\sqrt{\frac{12\,\kappa^2a_3-6\,\kappa\,a_2-2\,a_1-20\,a_3}{\alpha\,\kappa+\beta\,\kappa-b_1}}\operatorname{sech}(x+(2\kappa a_1+8\kappa^3a_3)t)e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2t+\theta_0)}. \quad (24)$$

The wave form outlined in (24) is represented by

$$(\alpha\,\kappa+\beta\,\kappa-b_1)(12\,\kappa^2a_3-6\,\kappa\,a_2-2\,a_1-20\,a_3)>0. \quad (25)$$

Result 3:

Employing (7), Eq. (18) changes to

$$\begin{cases} B_0=0, \quad B_1=\pm\sqrt{-\frac{12\,\kappa^2a_3-6\,\kappa\,a_2-2\,a_1-20\,a_3}{\alpha\,\kappa+\beta\,\kappa-b_1}}, \\ \omega=\kappa^4a_3-\kappa^3a_2-\kappa^2a_1-6\,\kappa^2a_3+3\,\kappa\,a_2+\sigma^2+a_1+a_3, \\ b_2=-\frac{6a_3(\alpha^2\kappa^2+2\,\alpha\,\beta\,\kappa^2+\beta^2\kappa^2-2\,\alpha\,\kappa\,b_1-2\,\beta\,\kappa\,b_1+{b_1}^2)}{36\,\kappa^4{a_3}^2-36\,\kappa^3a_2a_3-12\,\kappa^2a_1a_3+9\,\kappa^2{a_2}^2-120\,\kappa^2{a_3}^2+6\,\kappa\,a_1a_2+60\,\kappa\,a_2a_3+{a_1}^2+20\,a_1a_3+100\,{a_3}^2}. \end{cases} \quad (26)$$

Therefore, the solution for a singular soliton is

$$q(x,t)=\pm\sqrt{-\frac{12\,\kappa^2a_3-6\,\kappa\,a_2-2\,a_1-20\,a_3}{\alpha\,\kappa+\beta\,\kappa-b_1}}\operatorname{csch}(x+(2\kappa a_1+8\kappa^3a_3)t)e^{i(-\kappa x+\omega t+\sigma W(t)-\sigma^2t+\theta_0)}. \quad (27)$$

The wave form illustrated in (27) is given by

$$(\alpha\,\kappa+\beta\,\kappa-b_1)(12\,\kappa^2a_3-6\,\kappa\,a_2-2\,a_1-20\,a_3)<0. \quad (28)$$

Result 4:

According to (7), we can rewrite Eq. (18) as

$$\begin{cases} B_0=0, \quad B_1=\pm\sqrt{-\frac{6\,\kappa^2a_3-3\,\kappa\,a_2-a_1+5\,a_3}{2\,\alpha\,\kappa+2\,\beta\,\kappa-2\,b_1}}, \\ \omega=\kappa^4a_3-\kappa^3a_2-\kappa^2a_1+3\,\kappa^2a_3-3/2\,\kappa\,a_2+\sigma^2-1/2\,a_1+a_3, \\ b_2=-\frac{6a_3(\alpha^2\kappa^2+2\alpha\beta\kappa^2+\beta^2\kappa^2-2\alpha\kappa b_1-2\beta\kappa b_1+{b_1}^2)}{36\kappa^4{a_3}^2-36\kappa^3a_2a_3-12\kappa^2a_1a_3+9\kappa^2{a_2}^2+60\kappa^2{a_3}^2+6\kappa a_1a_2-30\kappa a_2a_3+{a_1}^2-10a_1a_3+25{a_3}^2}. \end{cases} \quad (29)$$

In this case, straddled singular–singular soliton solution is given by

$$q(x,t) = \pm\sqrt{-\frac{6\,\kappa^2 a_3 - 3\,\kappa\,a_2 - a_1 + 5\,a_3}{2\,\alpha\,\kappa + 2\,\beta\,\kappa - 2\,b_1}}\begin{Bmatrix}\coth(x + (2\kappa a_1 + 8\kappa^3 a_3)t) \\ \pm\text{csch}(x + (2\kappa a_1 + 8\kappa^3 a_3)t)\end{Bmatrix} e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0)}. \quad (30)$$

Furthermore, the complexiton solution is represented by

$$q(x,t) = \pm\sqrt{-\frac{6\,\kappa^2 a_3 - 3\,\kappa\,a_2 - a_1 + 5\,a_3}{2\,\alpha\,\kappa + 2\,\beta\,\kappa - 2\,b_1}}\begin{Bmatrix}\tanh(x + (2\kappa a_1 + 8\kappa^3 a_3)t) \\ \pm i\,\text{sech}(x + (2\kappa a_1 + 8\kappa^3 a_3)t)\end{Bmatrix} e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0)}. \quad (31)$$

Finally, the solution for a straddled dark-bright soliton is

$$q(x,t) = \pm\sqrt{-\frac{6\,\kappa^2 a_3 - 3\,\kappa\,a_2 - a_1 + 5\,a_3}{2\,\alpha\,\kappa + 2\,\beta\,\kappa - 2\,b_1}}\left\{\frac{\tanh(x + (2\kappa a_1 + 8\kappa^3 a_3)t)}{1 \pm \text{sech}(x + (2\kappa a_1 + 8\kappa^3 a_3)t)}\right\} e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0)}. \quad (32)$$

The wave forms referenced in (30)–(32) can be provided by

$$(\alpha\,\kappa + \beta\,\kappa - b_1)(6\,\kappa^2 a_3 - 3\,\kappa\,a_2 - a_1 + 5\,a_3) < 0. \quad (33)$$

4. Conclusions

In this study, we have successfully addressed dispersive optical solitons incorporating multiplicative white noise. Utilizing the *F*-expansion procedure, we have derived a variety of soliton solutions, including dark soliton solutions, singular soliton solutions, bright soliton solutions, straddled singular-singular soliton solutions, complexiton solutions, and straddled dark-bright soliton solutions. These solutions provide significant insights into the complex dynamics governed by the perturbed nonlinear wave equation under stochastic influences. We have also thoroughly discussed the parametric restrictions necessary for the existence of these soliton solutions. These constraints are crucial for the stability and realization of different soliton types in practical scenarios. Our analysis highlights the sensitivity of soliton behavior to the parameters involved, offering a deeper understanding of how perturbations and noise impact soliton dynamics. The findings of this paper contribute to the broader field of nonlinear wave equations, particularly in contexts where stochastic effects play a significant role. Future work could further explore the numerical simulations of these soliton solutions, investigate their stability under various perturbations, and extend the methods to other types of nonlinear equations with stochastic influences [1–10]. This study lays a groundwork for such explorations and opens up new avenues for research in the interplay between noise and nonlinear wave dynamics.

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Дисперсійне оптичне солітонне збурення з мультиплікативним білим шумом, що має параболічний закон самомодуляції фази

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Анотація. У цій статті ми досліджуємо дисперсійні оптичні солітони, що містять мультиплікативний білий шум. Використовуючи процедуру *F*-розкладення, ми отримуємо різні солітонні розв'язки, включаючи темні солітонні розв'язки, сингулярні солітонні розв'язки, яскраві солітонні розв'язки, розсіяні сингулярно-сингулярні солітонні розв'язки, комплексітонні розв'язки та розсіяні темно-яскраві солітонні розв'язки. Крім того, ми обговорюємо параметричні обмеження, необхідні для існування таких солітонних розв'язків, надаючи комплексний аналіз умов, за яких ці розв'язки є справедливими. Наші висновки сприяють розумінню динаміки збурених нелінійних хвильових рівнянь за наявності стохастичних впливів.

Ключові слова: солітони, білий шум, *F*-розкладення.