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Optics

Optical solitons with Kudryashov's form of local and non-local self-phase modulation structure having fractional temporal evolution

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Abstract. The current paper recovers optical soliton solutions with Kudrashov's proposed self-phase modulation structure and with fractional temporal evolution. This model could control and mitigate the Internet bottleneck effect, a severe hindrance to Internet traffic flow across intercontinental distances. Two independent integration approaches have made this retrieval possible. A wide spectrum of soliton solutions emerged with the collective applications of the integration schemes. The parameter constraints for the existence of these solitons are also presented.

Keywords: solitons, Internet bottleneck, integrability.

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1. Introduction

Optical solitons have made gigantic advances since their first inception in the telecommunications industry more than half a century ago. These solitons serve as bit carriers in optical fibers for transcontinental and transoceanic distances across the globe. Today's total dependence on internet communications across the globe would not have been possible without the unique engineering marvel of soliton science and technology. Therefore, it is imperative to address this technology from all angles to achieve its performance enhancement. One of the problems that has plagued the telecommunication industry is the Internet bottleneck effect [1–5]. This problem needs to be addressed and tackled with efficiency. Several mathematical measures and means have been proposed to address this unwanted issue [6–10].

One of them is to include spatio-temporal dispersion in addition to the pre-existing chromatic dispersion (CD) [11–15]. This can slow down the solitons across a junction point, and the traffic flow of signals can be monitored uniformly, like street traffic flow. Another measure that has been adopted is the consideration of the time-dependent coefficient of CD and self-phase modulation (SPM) [1]. This can also slow down the soliton, and in its limit, the solitons can be made to halt, allowing the cross-traffic internet to flow smoothly. The traffic light effect can similarly be adopted here as well. The third mechanism is based on using the fractional temporal evolution, which can slow down the evolution of pulses along one direction, and thus the traffic light effect can be implemented to control the bottleneck effect [16–20]. The current paper addresses this third measure to study the slow evolution of solitons for the governing nonlinear Schrödinger equation (NLSE) with fractional temporal evolution and linear CD. However, the SPM structure stems from the combination of Kudryashov's quadruple nonlinear effect and dual-form of nonlocal nonlinearity [21–24]. The nonlinearity parameter dictates the arbitrary intensity of pulses. The details of the model and the derivation of the soliton solutions are jotted, pictured, and exhibited in the rest of the paper after a succinct introduction to the model.

1.1. Governing model

This study investigates the time-fractional nonlinear Schrödinger equation, which includes Kudryashov's arbitrary refractive index alongside two distinct nonlocal nonlinearities [25]:

$$\frac{{}^{\mu}q}{t^{\mu}} + aq_{xx} + (c_1|q|^n + c_2|q|^{2n} + c_3|q|^{3n} + c_4|q|^{4n})q + (c_5(|q|^n)_{xx} + c_6(|q|^{2n})_{xx})q = 0,$$
$$0 < \mu \le 1, \tag{1}$$

where $q(x,t)$ represents the wave profile and $\frac{\partial^{\mu}(.)}{\partial t^{\mu}}$ denotes the conformable derivative, with n being the nonlinearity parameter. The coefficients c_i for $i = 0,1,2,3,4$ describe the nonlinearity effects, contributing to SPM. Furthermore, the coefficients c_5 and c_6 characterize a generalized non-local formulation of the refractive index. The power-law parameter n represents arbitrary intensity.

In this paper, we aim to implement two different approaches to improve existing techniques for constructing various soliton solutions dealing with NLSE. The modified simplest equation method is a highly effective and straightforward approach for finding solutions to nonlinear partial differential equations. Further, the research shows that this and the Kudryashov method are highly efficient, effective, impactful, straightforward, and easy to apply to various nonlinear partial differential equations.

Definition 1.1. *Let* $p: (0,\infty) \to R$. *The conformable derivative of order* β *can be introduced as follows:*

$$L_{\beta}(p)(x) = \lim_{a\to 0} \frac{p(x+ax^{1-\beta})-p(x)}{a}, \tag{2}$$

for all $x > 0$ and $\beta \in (0,1]$ [26].

2. Formulation of the problem

Here, we start with inserting the following wave transform into equation (1):

$$f(x,t) = U(\varsigma)e^{i\vartheta(x,t)},$$
$$\varsigma = x - v\frac{t^{\mu}}{\mu}, \vartheta(x,t) = u - kx + w\frac{t^{\mu}}{\mu}, \tag{3}$$

where u represents the phase center, k stands for the soliton frequency, and w denotes the wave frequency. Upon incorporating the transformations from equation (3) into equation (1), the resulting expression is as follows:

$$aU'' + 2ac_6nU^{2n}U'' + c_5nU^nU'' + +c_5n(n-1)U^{n-1}U'^2(2bk^2+w)U + +2c_6n(2n-1)U^{2n-1}U'^2 - c_1U^{n+1} - -c_2U^{2n+1} - c_4U^{4n+1} - c_3U^{3n+1} = 0, \tag{4}$$

and the imaginary part

$$(v + 2bk)U' = 0. \tag{5}$$

From the above equation, the velocity has the following form:

$$v = -2ak. \tag{6}$$

Consider

$$U(\varsigma) = V^{\frac{1}{n}}(\varsigma). \tag{7}$$

Thus, from equations (4) and (7), we obtain

$$a(1-n)V'^2 + naVV'' + 2c_6n^2aV''^{V^3} + 2c_6n^2aV'^2V^2 + c_5n^2V'V^2 - n^2(2ak^2+w)V^2 + +c_1n^2V^3 + c_2n^2V^4 + c_3n^2V^5 + c_4n^2V^6 = 0. \tag{8}$$

3. Modified simplest equation method

In this section, we present several novel conformable optical soliton solutions for the current model, derived using the modified simplest equation method. We assume that the solution to equation (10) can be expressed as the following series.

$$V(\varsigma) = \sum_{i=0}^{M} f_iG(\varsigma)^i, \tag{9}$$

where $f_0, f_1, \ldots, f_M$ are unknown constants, and M is a balancing parameter. In equation (8), implementing the balancing principle between V^3V'' and V^6 leads to $M = 1$. Here, from equation (9) the following is obtained:

$$V(\varsigma) = f_0 + f_1G(\varsigma), \tag{10}$$

where $G(\varsigma)$ satisfies the following equation:

$$G(\varsigma)' = h_0 + G(\varsigma)^2. \tag{11}$$

Now, the solutions of equation (13) with parameter s can be defined as follows [27]:

If $h_0 < 0$, we get:

$$G_1(\varsigma) = -\sqrt{-h_0}\tanh\left(\sqrt{-h_0}(\varsigma + s)\right), \tag{12}$$

$$G_2(\varsigma) = -\sqrt{-h_0}\coth\left(\sqrt{-h_0}(\varsigma + s)\right), \tag{13}$$

$$G_3(\varsigma) = \sqrt{-h_0}\begin{pmatrix} -\tanh\left(2\sqrt{-h_0}(\varsigma + s)\right) \\ \pm i\,\mathrm{sech}\left(2\sqrt{-h_0}(\varsigma + s)\right) \end{pmatrix}, \tag{14}$$

$$G_4(\varsigma) = \sqrt{-h_0}\begin{pmatrix} -\coth\left(2\sqrt{-h_0}(\varsigma + l)\right) \\ \pm csch\left(2\sqrt{-h_0}(\varsigma + s)\right) \end{pmatrix}, \qquad (15)$$

$$G_5(\varsigma) = -\frac{\sqrt{-h_0}}{2}\begin{pmatrix} \tanh\left(\frac{\sqrt{-h_0}}{2}(\varsigma + s)\right) \\ \pm\coth\left(\frac{\sqrt{-h_0}}{2}(\varsigma + s)\right) \end{pmatrix}. \qquad (16)$$

Substituting equations (10) and (11) into equation (8) yields a polynomial expressed in terms of powers of $G(\varsigma)$. We then arrange terms with similar powers and set each corresponding coefficient to zero. This process generates the following system of algebraic equations:

$$(G(\varsigma))^0: -ak^2n^2f_0^2 - n^2wf_0^2 + n^2c_1f_0^3 + n^2c_2f_0^4 + n^2c_3f_0^5 + n^2c_4f_0^6 + af_1^2h_0^2 - -anf_1^2h_0^2 + 2n^2c_6f_0^2f_1^2h_0^2 = 0, \qquad (17)$$

$$(G(\varsigma))^1: -2ak^2n^2f_0f_1 - 2n^2wf_0f_1 + 3n^2c_1f_0^2f_1 + +4n^2c_2f_0^3f_1 + 5n^2c_3f_0^4f_1 + 6n^2c_4f_0^5f_1 + 2anf_0f_1h_0 +2n^2c_5f_0^2f_1h_0 + 4n^2c_6f_0^3f_1h_0 + 4n^2c_6f_0f_1^3h_0^2 = 0, \qquad (18)$$

$$(G(\varsigma))^2: -ak^2n^2f_1^2 - n^2wf_1^2 + 3n^2c_1f_0f_1^2 + 6n^2c_2f_0^2f_1^2 + 10n^2c_3f_0^3f_1^2 + 15n^2c_4f_0^4f_1^2 + 2af_1^2h_0 + +4n^2c_5f_0f_1^2h_0 + 16n^2c_6f_0^2f_1^2h_0 + 2n^2c_6f_1^4h_0^2 = 0, \qquad (19)$$

$$(G(\varsigma))^3: 2anf_0f_1 + 2n^2c_5f_0^2f_1 + 4n^2c_6f_0^3f_1 + 4n^2c_2f_0f_1^3 + 10n^2c_3f_0^2f_1^3 + 20n^2c_4f_0^3f_1^3 + 2n^2c_5f_1^3h_0 + 20n^2c_6f_0f_1^3h_0 + n^2c_1f_1^3 = 0, \qquad (20)$$

$$(G(\varsigma))^5: 2n^2c_5f_1^3 + 16n^2c_6f_0f_1^3 + n^2c_3f_1^5 + 6n^2c_4f_0f_1^5 = 0 \qquad (21)$$

$$(G(\varsigma))^6: 6n^2c_6f_1^4 + n^2c_4f_1^6 = 0. \qquad (22)$$

The following results are obtained *via* solving this system:

Result 1.

$$f_0 = -\frac{2a(2+n)h_0}{n^2(c_1 - 4c_5h_0)}, f_1 = -\frac{2a(2+n)\sqrt{-h_0}}{\sqrt{n^4(c_1 - 4c_5h_0)^2}},$$
$$c_4 = \frac{3n^4c_6(c_1 - 4c_5h_0)^2}{2a^2(2+n)^2h_0},$$
$$c_2 = \frac{\begin{pmatrix} n^2(1+n)c_1^2 + 4n^2(4+n)c_1c_5h_0 + \\ 16(-n^2(5+2n)c_5^2 + 4a(2+n)^2c_6)h_0^2 \end{pmatrix}}{4a(2+n)^2h_0},$$
$$c_3 = \frac{\begin{bmatrix} n^2(c_1 - 4c_5h_0)\times \\ (n^2c_1c_5 + 4(-n^2c_5^2 + 5a(2+n)c_6)h_0) \end{bmatrix}}{2a^2(2+n)^2h_0},$$
$$w = -ak^2 - \frac{4ah_0}{n^2}. \qquad (23)$$

Result 2.

$$f_0 = \frac{\sqrt{2h_0(c_3c_5 + 50c_6^2h_0)} - 10c_6h_0}{c_3}, f_1 = \frac{\pm 1}{\sqrt{c_3}}\times$$
$$\times\sqrt{\frac{20c_6\left(\sqrt{2h_0(c_3c_5 + 50c_6^2h_0)} - 10c_6h_0\right)}{c_3} - 2c_5},$$
$$k = \frac{\sqrt{-n^2w - 4ah_0}}{\sqrt{an}},$$
$$c_1 = 4c_5h_0 - \frac{a(2+n)\begin{pmatrix} 10c_6h_0 + \\ \sqrt{2}\sqrt{h_0(c_3c_5 + 50c_6^2h_0)} \end{pmatrix}}{n^2c_5},$$
$$c_2 = \frac{a(1+n)\left(c_3c_5 + 10c_6\begin{pmatrix} 10c_6h_0 + \\ \sqrt{2h_0(c_3c_5 + 50c_6^2h_0)} \end{pmatrix}\right)}{2n^2c_5^2}$$
$$-14c_6h_0 - \sqrt{18h_0(c_3c_5 + 50c_6^2h_0)},$$
$$c_4 = \frac{3c_6\left(c_3c_5 + 10c_6\begin{pmatrix} 10c_6h_0 + \\ \sqrt{2}\sqrt{h_0(c_3c_5 + 50c_6^2h_0)} \end{pmatrix}\right)}{c_5^2}. \qquad (24)$$

Result 3.

$$f_1 = \pm\frac{\sqrt{2a(5+2n)f_0 - 32n^2c_6f_0^3}}{\sqrt{n^2(3c_1 + 2c_2f_0)}},$$
$$w = \frac{a(-ak^2(5+2n) + 6c_1f_0 + 4(c_2 + 4k^2n^2c_6)f_0^2)}{a(5+2n) - 16n^2c_6f_0^2},$$
$$c_3 = \frac{\begin{pmatrix} c_1(a + an - 44n^2c_6f_0^2) + \\ 2c_2f_0(a(2+n) - 20n^2c_6f_0^2) \end{pmatrix}}{-2a(5+2n)f_0^2 + 32n^2c_6f_0^4},$$
$$c_4 = \frac{3n^2c_6(3c_1 + 2c_2f_0)}{16n^2c_6f_0^3 - a(5+2n)f_0},$$
$$c_5 = \frac{2a(2+n)c_2f_0 + c_1(a + an + 16n^2c_6f_0^2)}{2n^2f_0(3c_1 + 2c_2f_0)},$$
$$h_0 = -\frac{n^2f_0(3c_1 + 2c_2f_0)}{2a(5+2n) - 32n^2c_6f_0^2}. \qquad (25)$$

Substituting equations (10) and (11) into equation (8) yields a polynomial expressed in terms of powers of $G(\varsigma)$. We then arrange terms with similar powers and set each corresponding coefficient to zero. This process generates the following system of algebraic equations:

$$q_1(x,t) = e^{i\left(u - kx + \frac{t^\mu\left(-ak^2 - \frac{4ah_0}{n^2}\right)}{\mu}\right)}\times$$
$$\left(-\frac{2a(2+n)h_0}{n^2(c_1 - 4c_5h_0)} \pm \eta_1\tanh\left[\left(x + \frac{2akt^\mu}{\mu}\right)\sqrt{-h_0}\right]\right)^{\frac{1}{n}}, \qquad (26)$$

where $\eta_1 = \frac{2a(2+n)\sqrt{-h_0}\sqrt{-h_0}}{\sqrt{n^4(c_1 - 4c_5h_0)^2}}$

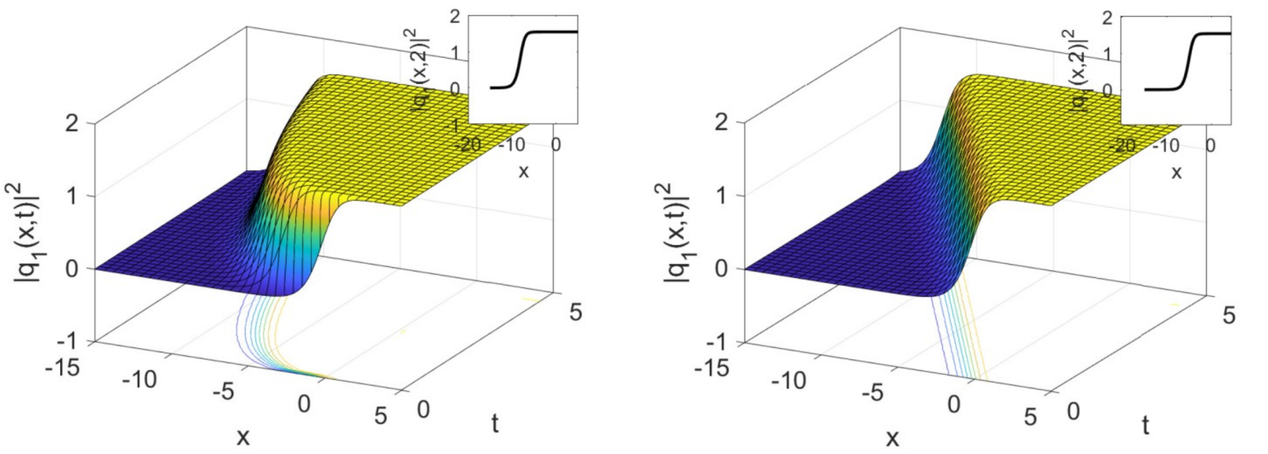


Fig. 1. The comparison of kink-type plots $|q_1(x,t)|^2$ for $\mu = 1$ and $\mu = 0.4$, where $n = 3$, $c_1 = 1, c_5 = 0.1$, $u = 0.1$, $a = 1.2$, $k = 1$, $h_0 = -1$, and $s = 1$.

$$q_2(x,t) = e^{i\left(u-kx+\frac{t^{\mu}\left(-ak^2-\frac{4ah_0}{n^2}\right)}{\mu}\right)} \times \left(-\frac{2a(2+n)h_0}{n^2(c_1-4c_5h_0)} \pm \eta_1 \coth\left[\left(s+x+\frac{2akt^{\mu}}{\mu}\right)\sqrt{-h_0}\right]\right)^{\frac{1}{n}}, \quad (27)$$

$$q_3(x,t) = e^{i\left(u-kx+\frac{t^{\mu}\left(-ak^2-\frac{4ah_0}{n^2}\right)}{\mu}\right)} \times \left(-\frac{2a(2+n)h_0}{n^2(c_1-4c_5h_0)} \pm \eta_1\left(i\operatorname{sech}\left[2\sqrt{-h_0}\varsigma\right] + \tanh\left[2\sqrt{-h_0}\varsigma\right]\right)\right)^{\frac{1}{n}}, \quad (28)$$

where $\varsigma = s + x + \frac{2akt^{\mu}}{\mu}$.

$$q_4(x,t) = e^{i\left(u-kx+\frac{t^{\mu}\left(-ak^2-\frac{4ah_0}{n^2}\right)}{\mu}\right)} \times \left(-\frac{2a(2+n)h_0}{n^2(c_1-4c_5h_0)} \pm \eta_1\left(csch\left[2\sqrt{-h_0}\varsigma\right] - \coth\left[2\sqrt{-h_0}\varsigma\right]\right)\right)^{\frac{1}{n}}, \quad (29)$$

$$q_5(x,t) = e^{i\left(u-kx+\frac{t^{\mu}\left(-ak^2-\frac{4ah_0}{n^2}\right)}{\mu}\right)} \times \left(-\frac{2a(2+n)h_0}{n^2(c_1-4c_5h_0)} \pm \eta_1\left(\coth\left[\frac{\sqrt{-h_0}\varsigma}{2}\right] + \tanh\left[\frac{\sqrt{-h_0}\varsigma}{2}\right]\right)\right)^{\frac{1}{n}} \quad (30)$$

Here, $\varsigma = s + x + \frac{2\sqrt{a}t^{\mu}\sqrt{-n^2w-4ah_0}}{n\mu}$

and

$$\eta_2 = \frac{\sqrt{-h_0}}{\sqrt{c_3}}\left(-2c_5 + \frac{20c_6\left(-10c_6h_0+\sqrt{2}\sqrt{h_0\left(c_3c_5+50c_6^2h_0\right)}\right)}{c_3}\right)^{\frac{1}{2}}. \quad (31)$$

$$q_6(x,t) = e^{i\left(u+\frac{wt^{\mu}}{\mu}-x\frac{\sqrt{(-n^2w-4ah_0)}}{\sqrt{a}n}\right)} \times \left(\frac{-10c_6h_0+\sqrt{-2h_0\left(c_3c_5+50c_6^2h_0\right)}}{c_3} \pm \eta_2\tanh\left[\sqrt{-h_0}\varsigma\right]\right)^{\frac{1}{n}}, \quad (32)$$

$$q_7(x,t) = e^{i\left(u+\frac{wt^{\mu}}{\mu}-\frac{x\sqrt{-n^2w-4ah_0}}{\sqrt{a}n}\right)} \times \left(\frac{-10c_6h_0+\sqrt{-2h_0\left(c_3c_5+50c_6^2h_0\right)}}{c_3} \pm \eta_2\coth\left[\sqrt{-h_0}\varsigma\right]\right)^{\frac{1}{n}}, \quad (33)$$

$$q_8(x,t) = e^{i\left(u+\frac{wt^{\mu}}{\mu}-\frac{x\sqrt{-n^2w-4ah_0}}{\sqrt{a}n}\right)} \times \left(f_0 \pm \eta_2\left(i\operatorname{sech}\left[2\sqrt{-h_0}\varsigma\right] + \tanh\left[2\sqrt{-h_0}\varsigma\right]\right)\right)^{\frac{1}{n}}, \quad (34)$$

$$q_9(x,t) = e^{i\left(u+\frac{wt^{\mu}}{\mu}-\frac{x\sqrt{-n^2w-4ah_0}}{\sqrt{a}n}\right)} \times \left(f_0 \pm \eta_2\left(-\coth\left[2\sqrt{-h_0}\varsigma\right] + csch\left[2\sqrt{-h_0}\varsigma\right]\right)\right)^{\frac{1}{n}}, \quad (35)$$

$$q_{10}(x,t) = e^{i\left(u+\frac{wt^{\mu}}{\mu}-\frac{x\sqrt{-n^2w-4ah_0}}{\sqrt{a}n}\right)} \times \left(f_0 \pm \frac{\eta_2}{2}\left(\coth\left[\frac{\sqrt{-h_0}}{2}\varsigma\right] + \tanh\left[\frac{\sqrt{-h_0}}{2}\varsigma\right]\right)\right)^{\frac{1}{n}}, \quad (36)$$

$$q_{11}(x,t) = e^{i\left(u-kx+\frac{at^{\mu}\left(-ak^2(5+2n)+6c_1f_0+4\left(c_2+4k^2n^2c_6\right)f_0^2\right)}{\mu\left(a(5+2n)-16n^2c_6f_0^2\right)}\right)} \times \left(f_0 \pm \eta_3\tanh[H_1]\right)^{\frac{1}{n}}, \quad (37)$$

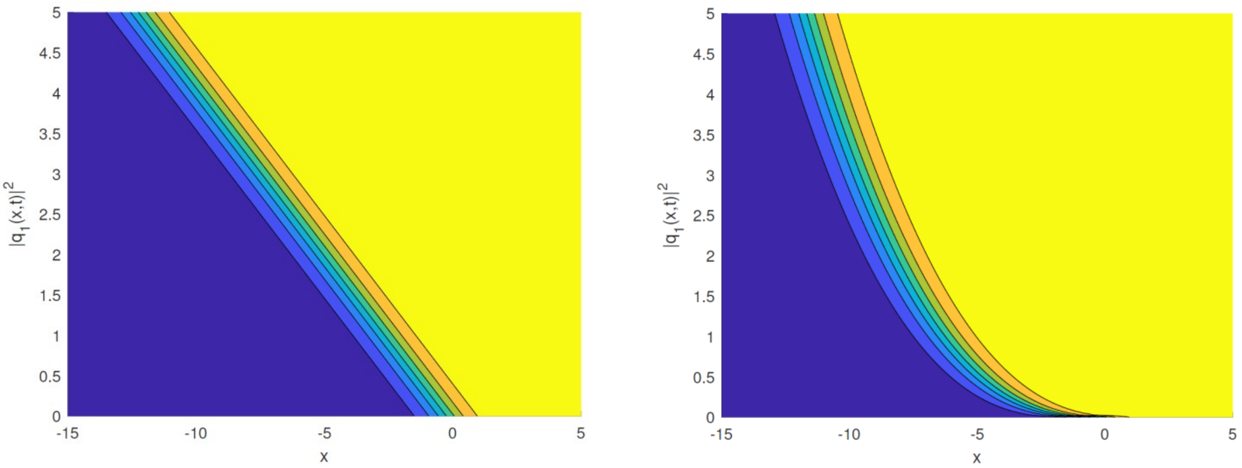


Fig. 2. The comparison of contour plots $Re(q_1(x,t))$ for $\mu = 1$ and $\mu = 0.4$, where $n = 3$, $c_1 = 1$, $c_5 = 0.1$, $u = 0.1$, $a = 1.2$, $k = 1$, $h_0 = -1$, and $s = 1$.

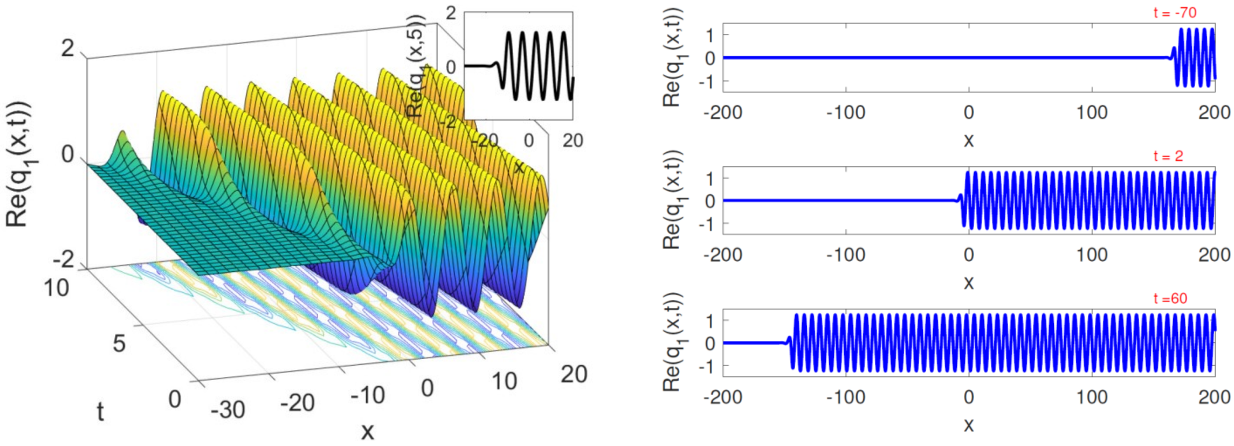


Fig. 3. The wave plots of $|q_1(x,t)|^2$ with the effect of temporal time parameter, where $n = 3, c_1 = 1, c_5 = 0.1, u = 0.1$, $a = 1.2, k = 1, h_0 = -1, \mu = 1$, and $s = 1$.

where $H_1 = \left(s + x + \frac{2akt^\mu}{\mu}\right)\sqrt{\frac{n^2 f_0(3c_1+2c_2f_0)}{2a(5+2n)-32n^2c_6f_0^2}}$ and

$$\eta_3 = \sqrt{\frac{n^2 f_0(3c_1+2c_2f_0)(2a(5+2n)f_0 - 32n^2c_6f_0^3)}{(2a(5+2n)-32n^2c_6f_0^2)(n^2(3c_1+2c_2f_0))}}.$$

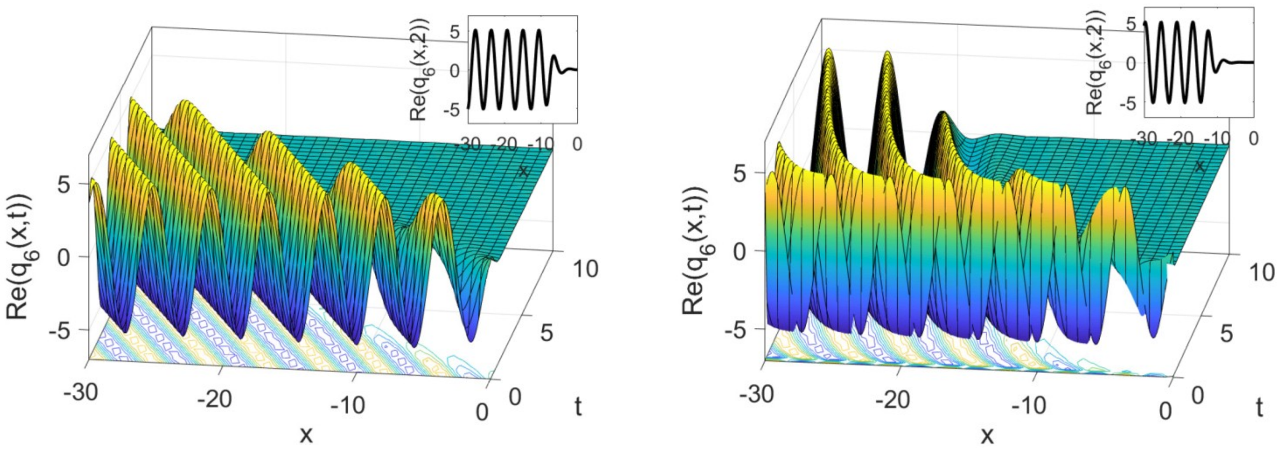
$$q_{12}(x,t) = e^{i\left(u-kx+\frac{at^\mu(-ak^2(5+2n)+6c_1f_0+4(c_2+4k^2n^2c_6)f_0^2)}{\mu(a(5+2n)-16n^2c_6f_0^2)}\right)}, \quad (f_0 \pm \eta_3\coth[H_1])^{\frac{1}{n}}, \tag{38}$$

$$q_{13}(x,t) = e^{i\left(u-kx+\frac{at^\mu(-ak^2(5+2n)+6c_1f_0+4(c_2+4k^2n^2c_6)f_0^2)}{\mu(a(5+2n)-16n^2c_6f_0^2)}\right)} (f_0 \pm \eta_3(\mathrm{isech}[2H_1] + \tanh[2H_1]))^{\frac{1}{n}}, \tag{39}$$

$$q_{14}(x,t) = e^{i\left(u-kx+\frac{at^\mu(-ak^2(5+2n)+6c_1f_0+4(c_2+4k^2n^2c_6)f_0^2)}{\mu(a(5+2n)-16n^2c_6f_0^2)}\right)} (f_0 \pm \eta_3(-\coth[2H_1] + csch[2H_1]))^{\frac{1}{n}}, \tag{40}$$

$$q_{15}(x,t) = e^{i\left(u-kx+\frac{at^\mu(-ak^2(5+2n)+6c_1f_0+4(c_2+4k^2n^2c_6)f_0^2)}{\mu(a(5+2n)-16n^2c_6f_0^2)}\right)} \left(f_0 \pm \eta_3\left(\coth\left[\frac{H_1}{2}\right] + \tanh\left[\frac{H_1}{2}\right]\right)\right)^{\frac{1}{n}}. \tag{41}$$

For completeness, we provide graphical simulations of the obtained solutions. The solution (26) is illustrated in Figs. 1–3 and 8 (kink-type, contour, wave and bright-profile comparisons). The solution (32) is displayed in Figs. 4 and 5 (wave and contour plots).



(a) (b)

Fig. 4. The comparison of wave plots of $Re\big(q_6(x,t)\big)$ for $\mu = 1$ and $\mu = 0.4$, where $n = 3$, $c_3 = -0.3$, $c_5 = 0.1$, $c_5 = 1$, $u = 0.1, a = 1.2$, $w = -2$, $h_0 = -1$, $\mu = 1$, and $s = 1$.

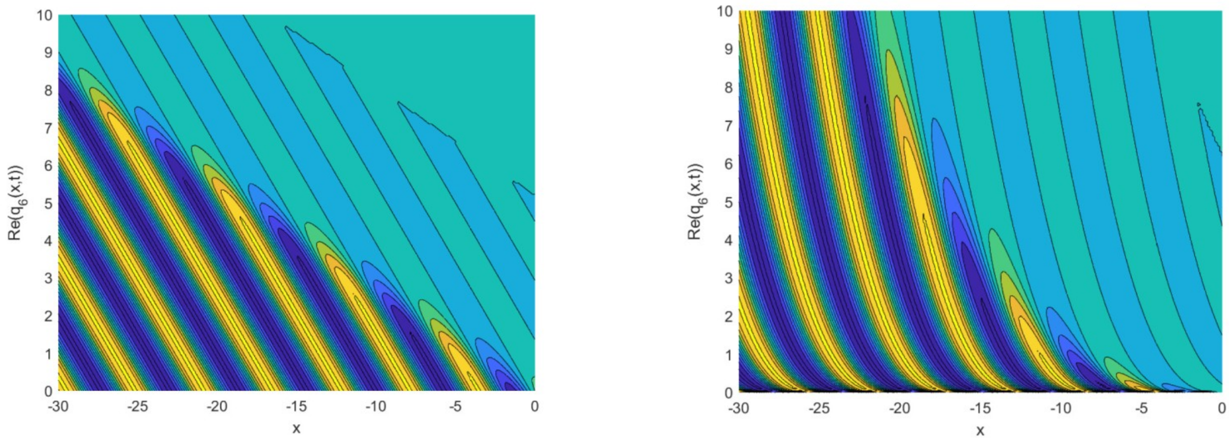


Fig. 5. The comparison of contour plots of $Re(q_6(x,t))$ for $\mu = 1$ and $\mu = 0.4$, where $n = 3, c_3 = -0.3, c_5 = 0.1, c_5 = 1, u = 0.1$, $a = 1.2, w = -2, h_0 = -1, \mu = 1$, and $s = 1$.

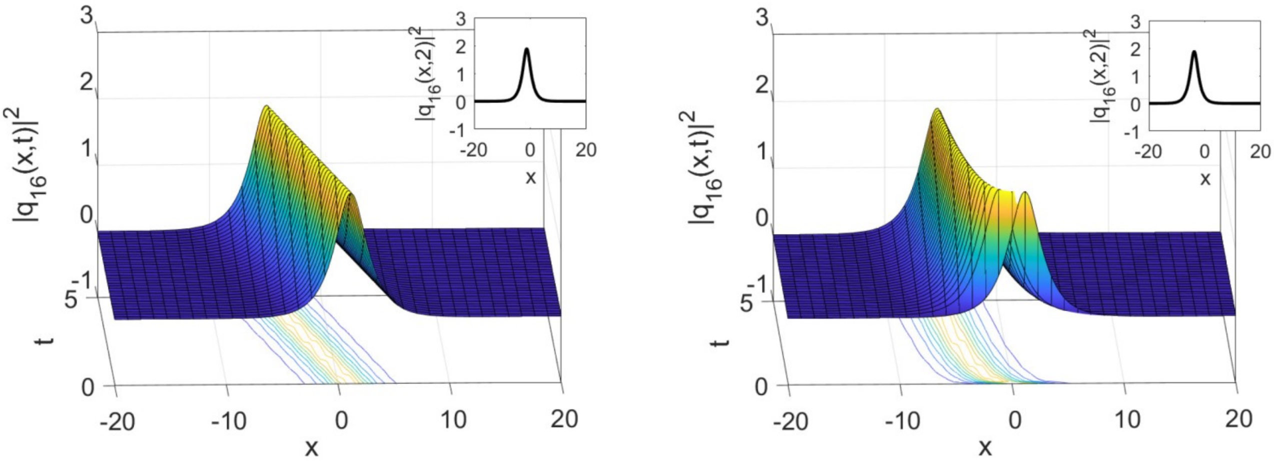
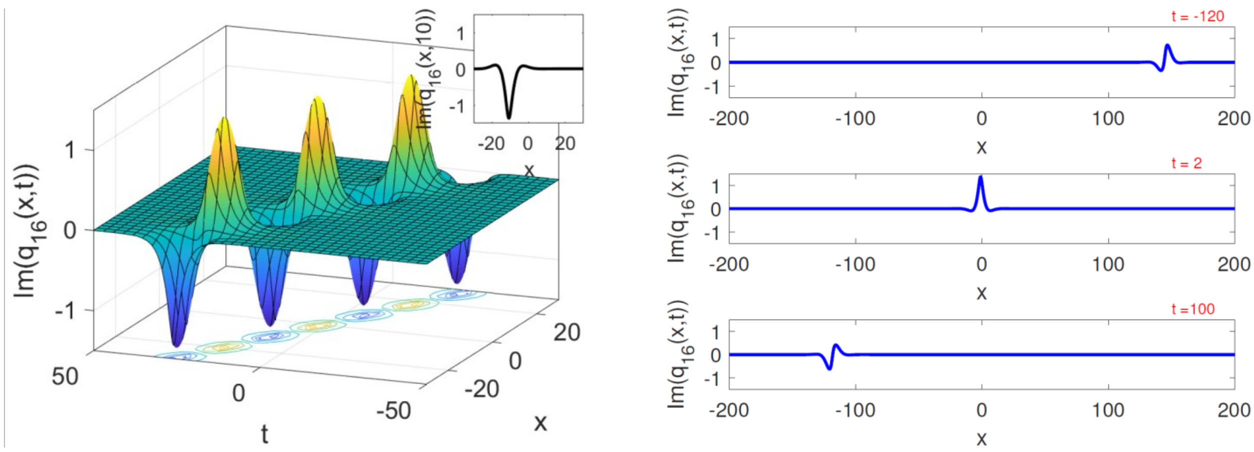
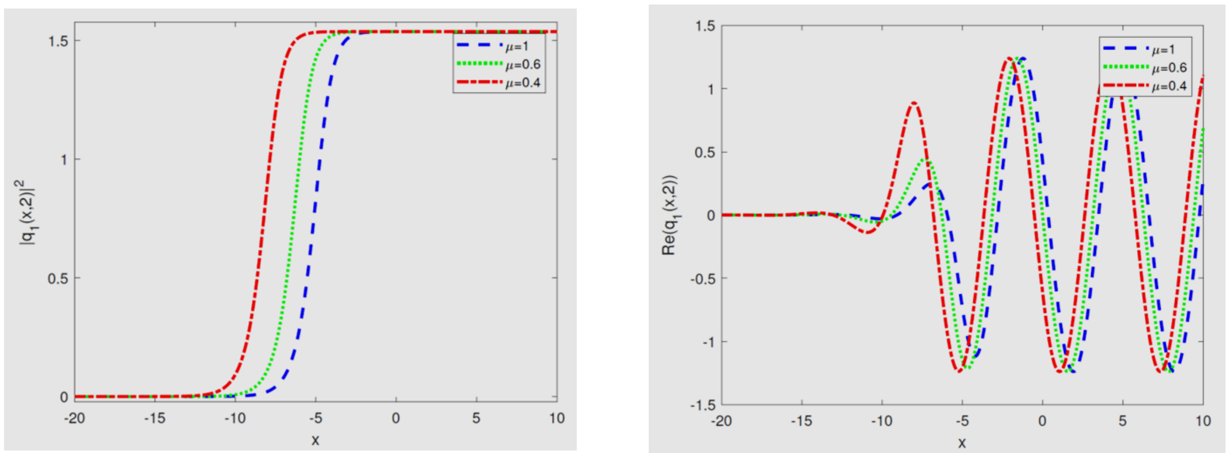


Fig. 6. The comparison of bright plots of $|q_{16}(x,t)|^2$ with the effect of different temporal parameter, where $c_5 = 1, n = 3$, $u = 0.1, a = 2, k = 0.3, c_3 = 0.3, s = 0.2, A = 0.1, B = 1$.



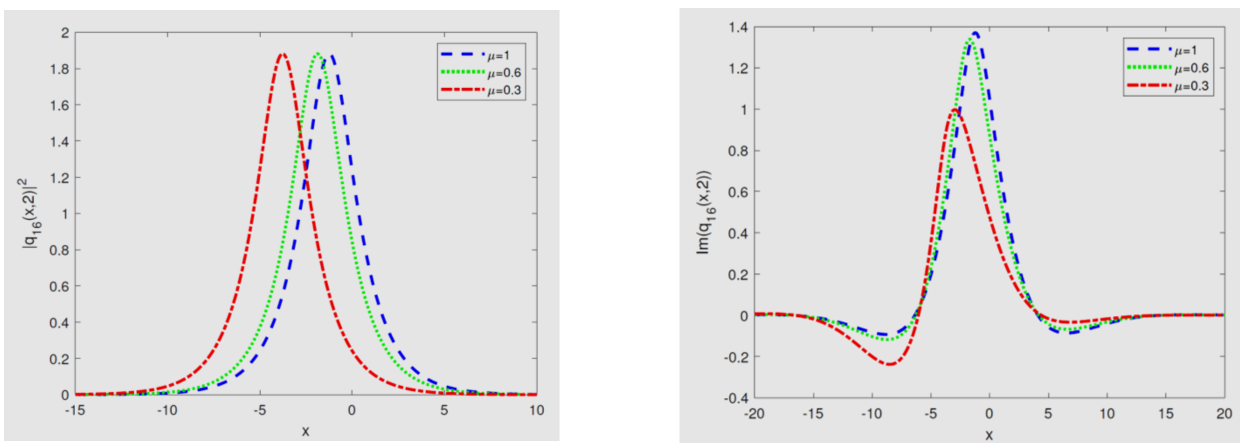
(a) (b)

Fig. 7. The dark-bright plots of $Im\left(q_{16}(x,t)\right)$ with the effect of the temporal parameter, where $c_5 = 1, n = 3, u = 0.1, a = 2, k = 0.3,\ c_3 = 0.3, s = 0.2, \mu = 1, A = 0.1, B = 1$.



(a) (b)

Fig. 8. The comparison of bright plots $|q_1(x,t)|^2$ and $Re(q_1(x,t))$ for $\alpha = 1$ and $\alpha = 0.4$, where $n = 3, c_1 = 1, c_5 = 0.1, u = 0.1, a = 1.2, k = 1, h_0 = -1$, and $s = 1$.



(a) (b)

Fig. 9. The effect of the conformable parameter on $|u_{17}(x,t)|^2$ and $Re\left(u_{17}(x,t)\right)$, where $c_5 = 1, n = 3, u = 0.1, a = 2, k = 0.3,\ c_3 = 0.3, s = 0.2, A = 0.1, B = 1$.

4. Kudryashov's approach

In this section, several new conformable optical soliton solutions to the current model are constructed using the Kudryashov method. The method relies on the following function [28]:

$$K(\varsigma) = \frac{1}{(A-B)\sinh(\varsigma)+(A+B)\cosh(\varsigma)}, \quad (42)$$

where A and B are non-zero constants, and the functions $K(\varsigma)$ satisfies the following relation:

$$(K'(\varsigma))^2 - K^2(\varsigma)(1-4ABK(\varsigma)^2) = 0. \quad (43)$$

According to the present method, the following series, based on the base function (42), is the solution of the proposed model:

$$V(\varsigma) = \sum_{i=0}^{N} f_i (K(\varsigma))^i, f_N \neq 0. \quad (44)$$

where the values $f_0, f_1, f_2, \ldots$ are the constants that need to be found, and N is the balancing value. Hence, the equation (44) has the following form:

$$V(\varsigma) = f_0 + f_1 G(\varsigma). \quad (45)$$

By substituting equation (45) and its derivatives, together with equation (3), into equation (8), we obtain a polynomial expression for $K(\varsigma)$. After gathering all the coefficients of $K(\varsigma)$ and equating them to zero, we form a system of algebraic equations. Solving this system yields the following results:

Result 1.

$$f_0 = 0, f_1 = \pm\frac{\sqrt{8ABc_5}}{\sqrt{c_3}}, w = a\left(\frac{1}{n^2} - k^2\right), \\ c_1 = -c_5, c_2 = \frac{a(1+n)c_3}{2n^2c_5} - \frac{4c_4c_5}{3c_3}, c_6 = \frac{c_4c_5}{3c_3}. \quad (46)$$

Using equations (3), (6), (7), (42), (45), and (46), we can have the following optical solutions:

$$q_{16}(x,t) = e^{i\left(u - kx + \frac{a\left(\frac{1}{n^2} - k^2\right)t^\mu}{\mu}\right)} \times \\ \left(-\frac{2^{\frac{3}{2}}\sqrt{ABc_5}}{((A+B)\cosh[\varsigma]+(A-B)\sinh[\varsigma])\sqrt{c_3}}\right)^{\frac{1}{n}}, \quad (47)$$

where $\varsigma = s + x + \frac{2akt^\mu}{\mu}$.

Plugging $A = B$ into the above solution, we obtain the following bright soliton solution

$$q_{17}(x,t) = e^{i\left(u - kx + \frac{a\left(-k^2 + \frac{1}{n^2}\right)t^\mu}{\mu}\right)} \times \\ \times \left(\pm\frac{sech\left[s + x + \frac{2akt^\mu}{\mu}\right]\sqrt{2c_5}}{\sqrt{c_3}}\right)^{\frac{1}{n}}. \quad (48)$$

Plugging $A = -B$ into the above solution, we obtain the following soliton solution

$$q_{18}(x,t) = e^{i\left(u - kx + \frac{a\left(-k^2 + \frac{1}{n^2}\right)t^\mu}{\mu}\right)} \times \\ \times \left(\pm\frac{\sqrt{-2B}csch\left[s + x + \frac{2akt^\mu}{\mu}\right]\sqrt{c_5}}{\sqrt{c_3B}}\right)^{\frac{1}{n}}. \quad (49)$$

Result 2.

$$f_0 = 0, f_1 = \pm\frac{\sqrt{8ABc_5}}{\sqrt{c_3}}, k = -\frac{\sqrt{a - n^2w}}{\sqrt{a}n}, \\ c_1 = -c_5, c_4 = \frac{3c_3(a(1+n)c_3 - 2n^2c_2c_5)}{8n^2c_5^2}, \\ c_6 = \frac{1}{8}\left(-2c_2 + \frac{a(1+n)c_3}{n^2c_5}\right). \quad (50)$$

Using equations (3), (6), (7), (42), (45), and (50), we can have the following optical solutions:

$$q_{19}(x,t) = e^{i\left(u + \frac{\sqrt{a-n^2w}x}{\sqrt{a}n} + \frac{t^\mu w}{\mu}\right)} \times \\ \left(\pm\frac{2^{\frac{3}{2}}\sqrt{ABc_5}}{((A+B)\cosh[\varsigma]+(A-B)\sinh[\varsigma])\sqrt{c_3}}\right)^{\frac{1}{n}}, \quad (51)$$

where $\varsigma = s + x - \frac{2\sqrt{a}t^\mu\sqrt{a-n^2w}}{n\mu}$.

Plugging $A = B$ into the above solution, we obtain the following bright soliton solution

$$q_{20}(x,t) = e^{i\left(u + \frac{\sqrt{a-n^2w}x}{\sqrt{a}n} + \frac{t^\mu w}{\mu}\right)} \times \\ \left(\pm\frac{\sqrt{2c_5}}{\sqrt{c_3}} sech\left[s + x - \frac{2\sqrt{a}t^\mu\sqrt{a-n^2w}}{n\mu}\right]\right)^{\frac{1}{n}}. \quad (52)$$

Plugging $A = -B$ into the above solution, we obtain the following soliton solution

$$q_{21}(x,t) = e^{i\left(u + \frac{\sqrt{a-n^2w}x}{\sqrt{a}n} + \frac{t^\mu w}{\mu}\right)} \times \\ \left(\pm\frac{\sqrt{-2Bc_5}}{\sqrt{Bc_3}} csch\left[s + x - \frac{2\sqrt{a}t^\mu\sqrt{a-n^2w}}{n\mu}\right]\right)^{\frac{1}{n}}. \quad (53)$$

Result 3.

$$f_0 = 0, f_1 = \pm\frac{\sqrt{-8ABc_1}}{\sqrt{c_3}}, k = -\frac{\sqrt{a-n^2w}}{\sqrt{a}n}, \\ c_5 = -c_1, c_4 = \frac{3c_3(2n^2c_1c_2 + a(1+n)c_3)}{8n^2c_1^2}, \\ c_6 = -\frac{c_2}{4} - \frac{a(1+n)c_3}{8n^2c_1}. \quad (54)$$

Using equations (3), (6), (7), (42), (45), and (54), we can have the following optical solutions:

$$q_{22}(x,t) = e^{i\left(u + \frac{\sqrt{a-n^2w}x}{\sqrt{a}n} + \frac{t^\mu w}{\mu}\right)} \times \\ \left(\pm\frac{2^{\frac{3}{2}}\sqrt{-ABc_1}}{((A+B)\cosh[\varsigma]+(A-B)\sinh[\varsigma])\sqrt{c_3}}\right)^{\frac{1}{n}}, \quad (55)$$

where $\varsigma = s + x - \frac{2\sqrt{a}t^\mu\sqrt{a-n^2w}}{n\mu}$.

Plugging $A = B$ into the above solution, we obtain the following bright soliton solution

$$q_{23}(x,t) = \mathrm{e}^{\mathrm{i}\left(u+\frac{\sqrt{a-n^2}wx}{\sqrt{a}n}+\frac{t^\mu w}{\mu}\right)} \times \left(\pm\frac{\sqrt{-2c_1}}{\sqrt{c_3}}\operatorname{sech}\left[x-\frac{2\sqrt{a}t^\mu\sqrt{a-n^2w}}{n\mu}\right]\right)^{\frac{1}{n}}. \quad (56)$$

Plugging $A = -B$ into the above solution, we obtain the following soliton solution

$$q_{24}(x,t) = \mathrm{e}^{\mathrm{i}\left(u+\frac{\sqrt{a-n^2}wx}{\sqrt{a}n}+\frac{t^\mu w}{\mu}\right)} \times \left(\frac{\sqrt{2Bc_1}}{\sqrt{Bc_3}}csch\left[x-\frac{2\sqrt{a}t^\mu\sqrt{a-n^2w}}{n\mu}\right]\right)^{\frac{1}{n}}. \quad (57)$$

The solution (47) is presented in Figs. 6 and 7 (bright and dark–bright profiles). Finally, Fig. 9 demonstrates the influence of the conformable parameter on solution (48).

6. Conclusions

This paper derived the optical soliton solutions to the governing NLSE with fractional temporal evolution, linear CD, and having Kudryashov's form of SPM structure. Two approaches have made this retrieval possible. They are the Kudryashov integration scheme and the modified simplest equation method. These two approaches collectively yielded the soliton solutions, which are enumerated and listed along with their respective parameter constraints that guarantee the existence of the solitons. The results are thus applicable to the telecommunications industry, which is the lifeline of modern-day internet communication, keeping the entire planet connected.

The future of this study stands on a very strong footing. The results will be later studied for additional form of fibers, namely polarization-preserving fibers, dispersion-flattened fibers, dispersion-managed fibers, *etc*. The model can also be applied to several forms of optoelectronic devices such as optical metamaterials and metasurfaces, optical couplers, magneto-optic waveguides, Bragg gratings, and others. The results of these research activities will be disseminated once they start pouring in sequentially.

Disclosures

Competing interests

The authors declare that they have no competing interests.

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Оптичні солітони форми Кудряшова із структурою локальної та нелокальної фазової самомодуляції з дробовою часовою еволюцією

M.A.S. Murad, F.K. Hamasalh, A.H. Arnous, A. Biswas, A.J.M. Jawad, Y. Yildirim, L. Moraru, C. Dragomir

Анотація. У цій статті відновлено оптичні солітонні розв'язки за допомогою запропонованої Кудряшовим структури фазової самомодуляції з дробовою часовою еволюцією. Ця модель може контролювати та пом'якшувати ефект вузького місця в мережі Інтернет, який є серйозною перешкодою для потоку інтернет-трафіку на міжконтинентальних відстанях. Два незалежні підходи до інтеграції зробили це відновлення можливим. Широкий спектр солітонних розв'язків з'явився завдяки колективному застосуванню схем інтеграції. Також представлено параметричні обмеження для існування таких солітонів.

Ключові слова: солітони, вузьке місце в мережі Інтернет, інтегрованість.