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Transport phenomena of two-dimensional band carriers with Dirac-like energetic spectrum

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Abstract. Conductivity of monolayer and bilayer graphene is considered with due regard for mutual drag of band electrons and holes. Search of contribution of the drag to conductivity shows that it sufficiently influences on mobility at high concentrations of carriers, which belong to different groups and have different drift velocities. In bilayer system the mutual drag can even change the direction of partial current. Magnetoresistivity and Hall effect were theoretically investigated for neutral and gated graphene. It is shown that for spatially unlimited neutral graphene Hall effect is totally absent. In gated, exactly monopolar graphene for the same case effect of magnetoresistivity vanishes; here the Hall constant does not involve any relaxation characteristic in contrast to results obtained for the popular method of τ-approximation. It is shown that limited sizes of crystal with monopolar conductivity can be cause for impressive dependence of the Hall constant and magnetoresistivity on the value of external magnetic field.

Keywords: graphene, quantum kinetic equation, conductivity, Hall effect, magnetoresistivity, mutual drag.

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1. Introduction

Great interest now exists to systems of 2D carriers with Dirac-like dispersion law (see, for instance, Refs. [1-6]):

$$\varepsilon^{(e)}(\vec{k}_\perp) = \hbar v_F k_\perp \,;\; \varepsilon^{(h)}(\vec{k}_\perp) = \hbar v_F k_\perp \,. \qquad (1.1)$$

For this case, the microscopic velocities are

$$\vec{v}^{(e,h)}(\vec{k}_\perp) = \hbar^{-1}\partial\varepsilon^{(e,h)}(\vec{k}_\perp)/\partial\vec{k}_\perp = v_F\vec{k}_\perp / k_\perp \,. \qquad (1.2)$$

The value of velocity v_F for graphene is about 10^6 m/s.

For neutral graphene ($\varepsilon_F = 0$) $n_e = n_h$. If Fermi level is shifted by applying to a gate some bias voltage V_a, we obtain the so-called *n*-graphene or *p*-graphene, where $\varepsilon_F = e|V_a|$. In this case (*a* = *e* or *h*),

$$n_a(V_a) = \frac{1}{\pi^2}\int f^{0(a)}_{\vec{k}\perp} d^2\vec{k}_\perp =$$

$$= \frac{1}{\pi^2}\int d^2\vec{k}_\perp \left[1+\exp\left(\frac{\varepsilon^{(a)}(k_\perp) - eV_a}{k_BT}\right)\right]^{-1} = \qquad (1.3)$$

$$= \frac{2}{\pi}\left(\frac{k_BT}{v_F\hbar}\right)^2 C(\kappa_a;1) \,.$$

Here, $\kappa_a = e|V_a|/k_BT$; $C(\kappa_a;u) = \int_0^\infty \frac{\kappa^u\, d\kappa}{1+\exp(\kappa-\kappa_a)}$; $(2/\pi)\,C(0;1) = 0.524$. At $\kappa_a >> 1$ (high level of degeneracy)

$$n_a = \pi^{-1}(e|V_a|/v_F\hbar)^2 = \pi^{-1}\left(k_F^{(a)}\right)^2 . \qquad (1.4)$$

2. Kinetic and balance equation

Consider here an uniform graphene crystal in constant uniform electrical and magnetic fields $\vec{E}$ and $\vec{H}$. For this case, the stationary quantum kinetic equation for the distribution function $f_{\vec{k}\perp}^{(a)}$ of band carriers from *a*-group, moving in the plane $z = 0$, can be presented in the form (see Ref. [7])

$$\frac{e_a}{\hbar}\vec{E}\frac{\partial f_{\vec{k}\perp}^{(a)}}{\partial \vec{k}_\perp} + \frac{e_a}{\hbar}\left\{\frac{1}{c}\left[\left(\vec{H}\times\frac{\partial}{\partial \vec{k}_\perp}\right), \vec{v}_\perp^{(a)}(\vec{k}_\perp)\right]_+ f_{\vec{k}\perp}^{(a)}\right\} = \mathrm{St}\, f_{\vec{k}\perp}^{(a)}.$$

(a = e or h). (2.1)

Here, $\mathrm{St}\, f_{\vec{k}\perp}^{(a)}$ is the collision integral. In this paper, we consider classical magnetic field (for massless fermions it corresponds to the following condition: $H << c(k_BT)^2/e\hbar v_F^2$). In what follows, we assume the following orientation of fields: $\vec{E} = \vec{E}_\perp = (E_x, E_y, 0)$; $\vec{H} = (0, 0, H_z)$.

Applying to both sides of Eq. (2.1) the operator $(1/\pi^2)\int \vec{k}_\perp d^2\vec{k}_\perp$, one obtains a set of exact balance equations for dynamic and statistic forces:

$$e_a[\vec{E}_\perp + (1/c)(\vec{H}\times\vec{u}_\perp^{(a)})] + \vec{F}_\perp^{(a,S)} + \sum_b \vec{F}_\perp^{(a,b)} = 0$$

(a = e or h). (2.2)

Here, $\vec{u}_\perp^{(a)} = \langle\vec{v}_{\vec{k}\perp}^{(a)}\rangle = (\pi^2 n_a)^{-1}\int \vec{v}_{\vec{k}\perp}^{(a)} f_{\vec{k}\perp}^{(a)})\, d^2\vec{k}_\perp$ is the drift velocity for carriers of *a*-type, the force $\vec{F}_\perp^{(a,S)}$ is connected with friction of drifting *a*-carriers with equilibrium external scattering system, the force $\vec{F}_\perp^{(a,b)}$ is connected with friction of drifting *a*-carriers with drifting *b*-carriers (intergroup drag);

$$\vec{F}_\perp^{(a,S)} = \frac{\hbar}{\pi^2 n_a}\int \vec{k}_\perp[\mathrm{St}^{(a,S)} f_{\vec{k}\perp}^{(a)}]\, d^2\vec{k}_\perp =$$

$$-\frac{e^2}{8\pi^4 n_a}\int d^2\vec{k}_\perp \int \vec{q}_\perp d^2\vec{q}_\perp \int d\omega\, \delta(\hbar\omega - \varepsilon_{\vec{k}\perp}^{(a)} + \varepsilon_{\vec{k}\perp-\vec{q}\perp}^{(a)})\times$$

$$\times\langle\varphi_{(S)}^2\rangle_{\omega,\vec{q}\perp}\{[f_{\vec{k}\perp}^{(a)}(1 - f_{\vec{k}\perp-\vec{q}\perp}^{(a)}) + f_{\vec{k}\perp-\vec{q}\perp}^{(a)}(1 - f_{\vec{k}\perp}^{(a)})]$$

$$\tanh(\hbar\omega/2k_BT) + f_{\vec{k}\perp}^{(a)} - f_{\vec{k}\perp-\vec{q}\perp}^{(a)}\}; \quad (2.3)$$

here $\langle\varphi^2\rangle_{\omega,\vec{q}\perp} = (2\pi)^{-1}\int_{-\infty}^{\infty}\langle\varphi^2\rangle_{\omega,\vec{q}} dq_z$, where φ is the fluctuating external scattering potential;

$$\vec{F}_\perp^{(a,b)} = (\hbar/\pi^2 n_a)\int \vec{k}_\perp[\mathrm{St}^{(a,b)} f_{\vec{k}\perp}^{(a)}]\, d^2\vec{k}_\perp =$$

$$= \frac{e^4\hbar}{4\pi^3\varepsilon_L^2 n_a}\int d^2\vec{k}_\perp \int d^2\vec{k}_\perp' \int d\omega \int \vec{q}_\perp \frac{d^2\vec{q}_\perp}{q_\perp^2}\delta(\hbar\omega -$$

$$-\varepsilon_{\vec{k}\perp}^{(a)} + \varepsilon_{\vec{k}\perp-\vec{q}\perp}^{(a)})\,\delta(\hbar\omega - \varepsilon_{\vec{k}'\perp}^{(b)} + \varepsilon_{\vec{k}'\perp-\vec{q}\perp}^{(b)})\times$$

$$\times\exp(-2q_\perp l_{ab})[f_{\vec{k}\perp-\vec{q}\perp}^{(a)}(1 - f_{\vec{k}\perp}^{(a)}) f_{\vec{k}'\perp}^{(b)}(1 - f_{\vec{k}'\perp-\vec{q}\perp}^{(b)}) -$$

$$- f_{\vec{k}'\perp-\vec{q}\perp}^{(b)}(1 - f_{\vec{k}'\perp}^{(b)}) f_{\vec{k}\perp}^{(a)}(1 - f_{\vec{k}\perp-\vec{q}\perp}^{(a)})]. \quad (2.4)$$

In formulae (2.3) and (2.4), one assumes contribution of two-dimensional carriers to screening as negligible in comparison with the dielectric constant ε_L of crystal lattice. The absence of divergence at the coulomb scattering of particles of two-dimensional gas allows to refuse from specific screening so necessary (but badly grounded) for three-dimensional charged gas. The factor $\exp(-2q_\perp l_{ab})$ under integral in (2.4) concerns that the case when carriers of *a*-type and *b*-type move in parallel layers, separated by space l_{ab}.

Designate the equilibrium distribution function of *a*-particles by the symbol $f_0(\varepsilon^{(a)}(\vec{v}_{\vec{k}\perp})) \equiv f_{\vec{k}\perp}^{0(a)}$. The following consideration in this article will be performed using the model non-equilibrium distribution function of the following form:

$$f_{\vec{k}\perp}^{(a)} = f_0\left(\varepsilon^{(a)}(\vec{v}_{\vec{k}\perp}^{(a)} - \vec{u}_\perp^{(a)})\right) = \left\{1 + \exp\left[\frac{\hbar(v_F k_\perp - \vec{k}_\perp\vec{u}_\perp^{(a)}) - eV_a}{k_BT}\right]\right\}^{-1}. \quad (2.5)$$

It contains the vector $\vec{u}_\perp^{(a)}$ as a model parameter. The meaning of the latter is evident after calculation of the averaged velocity of *a*-particles:

$$\langle\vec{v}_{\vec{k}\perp}^{(a)}\rangle = \frac{1}{\pi^2 n_a}\int \vec{v}_{\vec{k}\perp}^{(a)} f_{\vec{k}\perp}^{(a)}\, d^2\vec{k}_\perp =$$

$$= \frac{1}{\pi^2 n_a}\int \vec{v}_{\vec{k}\perp}^{(a)} f_0(\varepsilon_{\vec{k}\perp}^{(a)}(\vec{v}_{\vec{k}\perp}^{(a)} - \vec{u}_\perp^{(a)}))\, d^2\vec{k}_\perp = \vec{u}_\perp^{(a)}.$$

Thus, the vector $\vec{u}_\perp^{(a)}$ is the drift velocity for the group of *a*-particles. The density of two-dimensional current in a uniform crystal for the corresponding group (a = e and h) is $\vec{j}_a = e_a n_a \vec{u}_\perp^{(a)}$. The total density of current in graphene is $\vec{j} = -en_e\vec{u}_\perp^{(e)} + en_h\vec{u}_\perp^{(h)}$.

Substituting the model functions (2.5) in the formulae (2.3) and (2.4), one obtains:

$$\vec{F}_{\perp}^{(a,S)} = -\frac{2e^2}{(2\pi)^4 n_a}\int d^2\vec{k}_{\perp}\int \vec{q}_{\perp} d^2\vec{q}_{\perp}\times$$
$$\times\int d\omega\,\delta(\hbar\omega-\varepsilon^{(a)}_{\vec{k}_{\perp}}+\varepsilon^{(a)}_{\vec{k}_{\perp}-\vec{q}_{\perp}})\times$$
$$\times\int (f^{(a)}_{\vec{k}_{\perp}}-f^{(a)}_{\vec{k}_{\perp}-\vec{q}_{\perp}})\{\tanh(\hbar\omega/2k_BT)\times$$
$$\times\coth[\hbar(\omega-\vec{u}^{(a)}_{\perp}\vec{q}_{\perp})/2k_BT]-1\}\langle\varphi^{\,2}_{(S)}\rangle_{\omega,\vec{q}_{\perp}}; \quad (2.6)$$

$$\vec{F}_{\perp}^{(a,b)} = -\frac{2e^4\hbar}{(2\pi)^3\varepsilon_L^2 n_a}\int d^2\vec{k}_{\perp}\int d^2\vec{k}'_{\perp}\int d\omega\int \vec{q}_{\perp}\frac{d^2\vec{q}_{\perp}}{q_{\perp}^2}\,\delta\times$$
$$\times\left(\hbar\omega-\varepsilon^{(a)}_{\vec{k}_{\perp}}+\varepsilon^{(a)}_{\vec{k}_{\perp}-\vec{q}_{\perp}}\right)\delta\left(\hbar\omega-\varepsilon^{(b)}_{\vec{k}'_{\perp}}+\varepsilon^{(b)}_{\vec{k}'_{\perp}-\vec{q}_{\perp}}\right)\times$$
$$\times\exp(-2q_{\perp}l_{ab})\left[f^{(a)}_{\vec{k}_{\perp}-\vec{q}_{\perp}}\left(1-f^{(a)}_{\vec{k}_{\perp}}\right)f^{(b)}_{\vec{k}'_{\perp}}\left(1-f^{(b)}_{\vec{k}'_{\perp}-\vec{q}_{\perp}}\right)-\right.$$
$$\left.-f^{(b)}_{\vec{k}'_{\perp}-\vec{q}_{\perp}}\left(1-f^{(b)}_{\vec{k}'_{\perp}}\right)f^{(a)}_{\vec{k}_{\perp}}\left(1-f^{(a)}_{\vec{k}_{\perp}-\vec{q}_{\perp}}\right)\right]. \quad (2.7)$$

Introducing the function (2.5) into expressions (2.6) and (2.7) and performing linearization of forces $\vec{F}_{\perp}^{(a,S)}$ and $\vec{F}_{\perp}^{(a,b)}$ over drift velocities $\vec{u}_{\perp}^{(a)}$ and $\vec{u}_{\perp}^{(b)}$, we obtain the following convenient forms:

$$\vec{F}_{\perp}^{(a,S)} = -e_a\tilde{\beta}^{(a)}\vec{u}_{\perp}^{(a)};$$
$$\vec{F}_{\perp}^{(a,b)} = -e_a\xi^{(a,b)}(\vec{u}_{\perp}^{(a)}-\vec{u}_{\perp}^{(b)}). \quad (2.8)$$

Here, $(i, j = x, y)$

$$\beta^{(a)}_{ij} = \frac{e_a\hbar}{8\pi^4 k_B T\, n_a}\int d^2\vec{k}_{\perp}\int q_i q_j d^2\vec{q}\times$$
$$\times\int d\omega\,\delta\left(\hbar\omega-\varepsilon^{(a)}_{\vec{k}_{\perp}}+\varepsilon^{(a)}_{\vec{k}_{\perp}-\vec{q}_{\perp}}\right)\times$$
$$\times\left(f_0\left(\varepsilon^{(a)}_{\vec{k}_{\perp}}\right)-f_0\left(\varepsilon^{(a)}_{\vec{k}_{\perp}-\vec{q}_{\perp}}\right)\right)\times$$
$$\times\sinh^{-1}\left(\frac{\hbar\omega}{k_BT}\right)\langle\varphi^{\,2}_{(S)}\rangle_{\omega,\,\vec{q}_{\perp}}. \quad (2.9)$$

$$\xi^{(a,b)}_{ij} = \frac{e^2 e_a\hbar^2}{8\pi^3 k_B T\varepsilon_L^2 n_a}\int d^2\vec{k}_{\perp}\int d^2\vec{k}'_{\perp}\int\frac{q_i q_j}{q_{\perp}^2}d^2\vec{q}_{\perp}\times$$
$$\times\int d\omega\,\delta\left(\hbar\omega-\varepsilon^{(a)}_{\vec{k}_{\perp}}+\varepsilon^{(a)}_{\vec{k}_{\perp}-\vec{q}_{\varphi}}\right)\delta\left(\hbar\omega-\varepsilon^{(b)}_{\vec{k}'_{\perp}}+\varepsilon^{(b)}_{\vec{k}'_{\perp}-\vec{q}_{\varphi}}\right)\times$$
$$\times\exp(-2q_{\perp}l_{ab})\left(f_0\left(\varepsilon^{(a)}_{\vec{k}_{\perp}}\right)-f_0\left(\varepsilon^{(a)}_{\vec{k}_{\perp}-\vec{q}_{\varphi}}\right)\right)\left(f_0\left(\varepsilon^{(a)}_{\vec{k}_{\perp}}\right)-\right.$$
$$\left.-f_0\left(\varepsilon^{(a)}_{\vec{k}_{\perp}-\vec{q}_{\varphi}}\right)\right)\left(f_0\left(\varepsilon^{(b)}_{\vec{k}'_{\perp}}\right)-f_0\left(\varepsilon^{(b)}_{\vec{k}'_{\perp}-\vec{q}_{\varphi}}\right)\right)\sinh^{-2}\left(\frac{\hbar\omega}{2k_BT}\right). \quad (2.10)$$

For elastic and isotropic scattering $\langle\varphi^2\rangle_{\omega,\vec{q}_{\perp}} = \langle\varphi^2\rangle_{q_{\perp}}\delta(\omega)$. Then,

$$\beta^{(a)}_{ij} = \delta_{ij}\frac{e_a\hbar}{16\pi^4 n_a}\int d^2\vec{k}_{\perp}\int q_{\perp}^2 d^2\vec{q}_{\perp}\delta\times$$
$$\times\left(-\varepsilon^{(a)}_{\vec{k}_{\perp}}+\varepsilon^{(a)}_{\vec{k}_{\perp}-\vec{q}_{\perp}}\right)\frac{\partial f_0(\varepsilon^{(a)}_{\vec{k}_{\perp}})}{\partial\varepsilon^{(a)}_{\vec{k}_{\perp}}}\langle\varphi^{\,2}_{(S)}\rangle_{q_{\perp}}\equiv\delta_{ij}\beta^{(a)}. \quad (2.11)$$

Uniting Eqs. (2.2) and (2.8), we obtain the system of the following vector equations:

$$\vec{E}_{\perp}+(1/c)(\vec{H}\times\vec{u}_{\perp}^{(a)})-\tilde{\beta}^{(a)}\vec{u}_{\perp}^{(a)}-$$
$$-\sum_b\frac{e_b}{e_a}\tilde{\xi}^{(a,b)}(\vec{u}_{\perp}^{(a)}-\vec{u}_{\perp}^{(b)})=0 \quad (2.12)$$

3. Kinetic coefficients

To calculate the values $\beta^{(a)}$, we have at first to construct the correlator $\langle\varphi^{\,2}_{(S)}\rangle_{\vec{q}_{\perp}}$. For an external scattering system represented by neutral impurities with the concentration $n_{NI}^{(2)}$ (see Eqs. (2.11) and (*A*.1))

$$\beta^{(a)}_{(NI)} = \frac{e_a e^2 n_{NI}^{(2)}(k_BT)^2}{\varepsilon_L^2 q_B^2 v_F^4\hbar^3}D(\kappa_a). \quad (3.1)$$

Here, $D(\kappa_a) = 9\pi^2 C(\kappa_a;3)/[2C(\kappa_a;1)]$; $D(0)\approx 306.8$; $D(\kappa_a >> 1)\approx 9\pi^2\kappa_a^2/4$.

For charged impurities (see below (*A*.2))

$$\beta^{(a)}_{(CI)} = -2\pi^2 e_a e^2 n_{CI}^{(2)}/\varepsilon_L^2\hbar\, v_F^2. \quad (3.2)$$

Note that the expression (3.2) does not depend on the bias voltage V_a.

For the layer of impurities separated from graphene by the distance $l >> n_a^{-1/2}$, one has to introduce the factor $1/2\sqrt{\pi^3 n_a l}$ into Eqs. (3.1) and (3.2).

To calculate the term responsible for *a*-*b*-drag for carriers occupying the same plane, we have used the formula (2.10) at $l_{ab}=0$. After some simplification, we obtain

$$\xi^{(a,b)} = \frac{e_b n_b}{e_a n_b}\xi^{(b,a)}\approx$$
$$\approx\frac{9\pi e^2 e_a(k_BT)^2}{2\varepsilon_L^2\hbar^3 v_F^4}\frac{C(\kappa_a;0.5)C(\kappa_b;0.5)}{C(\kappa_a;1)}. \quad (3.3)$$

If *a*- and *b*-carriers occupy the planes separated by the distance l_{ab}, then for $l_{ab} >> (n_a n_b)^{-1/4}$ we have to multiply the expression (3.3) by the factor $1/2\sqrt{\pi^3}(n_a n_b)^{1/4} l_{ab}$.

4. Conductivity of graphene in absence of magnetic field

4.1. Neutral graphene

Here, we assume $V_e = V_h = 0$. Then, at $\vec{H} = 0$ one obtains from Eqs. (2.12):

$$\vec{E}_\perp + \beta\,\vec{u}_\perp^{(e)} + \xi(\vec{u}_\perp^{(e)} - \vec{u}_\perp^{(h)}) = 0\,,$$
$$\vec{E}_\perp - \beta\,\vec{u}_\perp^{(h)} - \xi(\vec{u}_\perp^{(h)} - \vec{u}_\perp^{(e)}) = 0\,, \qquad (4.1)$$

where (see (3.3)) $\xi \approx 10\, e^3 (k_B T)^2 / \varepsilon_L^2 \hbar^3 v_F^4$,

$$\beta_{(CI)} \approx 20\, e^3 n_{CI}^{(2)} / \varepsilon_L^2 \hbar\, v_F^2\,,$$
$$\beta_{(NI)} \approx 300 e^3 n_{NI}^{(2)} (k_B T)^2 / \varepsilon_L^2 q_B^2\, v_F^4 \hbar^3\,.$$

It follows from (4.1) that for negligible drag ($\xi \to 0$) the coefficient β is in inverse proportion to the mobility of carriers ($e > 0$). Solving the system (4.1), one finds:

$$\vec{u}_\perp^{(h)} = -\,\vec{u}_\perp^{(e)} = \vec{E}_\perp / (\beta + 2\xi) = \mu\,\vec{E}_\perp\,. \qquad (4.2)$$

The density of total current:

$$\vec{j}_\perp = \vec{j}_\perp^{(e)} + \vec{j}_\perp^{(h)} = \sigma_\perp \vec{E}_\perp = \frac{2en^{(e)}}{\beta + 2\xi}\vec{E}_\perp \approx$$
$$\approx \frac{e}{\beta + 2\xi}\left(\frac{k_B T}{\hbar v_F}\right)^2 \vec{E}_\perp\,. \qquad (4.3)$$

For scattering by neutral impurities, the mutual drag is negligible, if $n_{NI}^{(2)} >> q_B^2 / 20$; for charged impurities one can neglect the $e-h$-drag, if $n_{CI}^{(2)} >> (k_B T / \hbar v_F)^2$.

4.2. Monopolar graphene

Here, we consider *n*-graphene with high-degenerate carriers ($k_B T << eV_e$). For this case, the number of holes is very small, and we can neglect their contribution to conductivity. Let the external scattering system is represented by charged impurities belonging to the plane of graphene. It follows from Eqs. (2.12), (3.2):

$$\vec{E}_\perp + \beta^{(e)}\vec{u}_\perp^{(e)} = 0\,;$$
$$\beta^{(e)} = -\mu^{-1} = -2\pi^2 e^3 n_{CI}^{(2)} / \varepsilon_L^2 \hbar\, v_F^2\,. \qquad (4.4)$$

At $v_F = 10^8$ cm/s, $\varepsilon_L = 10$, $n_{CI}^{(2)} = 10^{11}\,\mathrm{cm}^{-2}$ one obtains: $\mu \approx 1.5 \cdot 10^4\,\mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$. This value does not depend on the gate-voltage V_e. So, the conductivity of monopolar graphene depends linearly on the carrier density, in accord with the experimental data (see Ref. [1]). For the considered case, the conductivity

$$\sigma_\perp = -en_e / \beta^{(e)} = en_e\mu = \varepsilon_L^2 V_e^2 / 2\pi^3 \hbar\, n_{CI}^{(2)}\,. \qquad (4.5)$$

So, at a gate bias the conductivity of graphene rises as the square of gate voltage: $\sigma_\perp \propto V_e^2$.

Let the external scattering system is represented by neutral impurities. Then the conductivity

$$\sigma_\perp = en_e\mu_e = -\,en_e / \beta^{(e)} =$$
$$= 8\varepsilon_L^2\, q_B^2 \hbar v_F^2 / 9\pi^4 e^2 n_{(NI)}^2\,. \qquad (4.6)$$

Thereof, for neutral scattering centres the conductivity does not depend on the voltage V_e.

5. Conductivity of two-layer graphene in absence of magnetic field

The two-layer graphene system is especially interesting. That contains two graphene layers separated by an ultrathin highly insulating dielectric layer with the width l (see, as a possible example, two-gate composition in Fig. 1). The carriers in graphene –1 and graphene –2 are accepted as high-degenerate $\left(k_B T << e\left|V_{1,2}\right|\right)$. Therefore, $n_{1,2} = \pi^{-1}\left(e\left|V_{1,2}\right| / v_F \hbar\right)^2$.

Represent the external scattering system by charged impurities disposed with equal densities in both graphene planes. As an example, consider here the most actual case $V_1 = V_e > 0$ and $V_2 = V_h > 0$; then graphene –1 has electron conductivity and graphene –2 has hole conductivity.

For the considered case, the system of equations (4.1) can be written in the following form:

$$E_{1x} - \beta^{(e)} u_x^{(e)} + \xi\left(u_x^{(e)} - u_x^{(h)}\right) = 0 \quad ; \qquad (5.1)$$
$$E_{2x} - \beta^{(h)} u_x^{(h)} + \xi\left(u_x^{(e)} - u_x^{(h)}\right)(n_e / n_h) =$$
$$= E_{2x} - \beta^{(h)} u_x^{(h)} + \xi\left(u_x^{(e)} - u_x^{(h)}\right)(V_e / V_h)^2 = 0\,. \qquad (5.2)$$

Here, $\beta^{(e)} < 0$, $\beta^{(h)} > 0$, $\xi > 0$ and in accordance with p. 4

$$\beta^{(a)} \approx \frac{2\pi^2 e^2 e_a n_{CI}^{(2)}}{\varepsilon_L^2 \hbar\, v_F^2}\left[1 + \frac{\hbar\, v_F}{2\pi e V_a l}\right],$$
$$\xi = -\,\xi^{(e,h)} \approx \frac{2e^3 k_B T}{9\varepsilon_L^2 \hbar^2 v_F^3 l}\frac{V_h}{V_e}\,. \qquad (5.3)$$

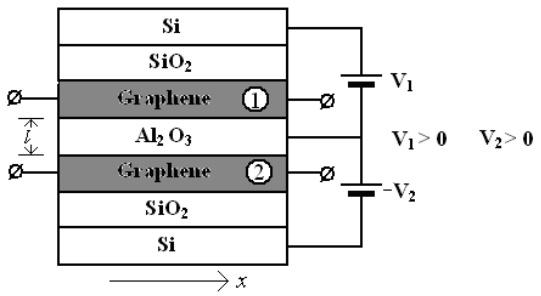

Fig. 1. Controlled bilayer graphene composition.

It follows from (5.3) and (3.3) that the drag coefficient for electrons in *n*-graphene linearly depends on the gate voltage applied to *p*-graphene (and vice verse).

The electric field components E_{1x} and E_{2x} can have the same signs as the opposite ones. If the inequality $eV_a l >> \hbar v_F$ is valid, then (see (5.3)) $-\beta^{(e)} = \beta^{(h)} = \beta = 2\pi^2 e^3 n_{CI}^{(2)} / \varepsilon_L^2 \hbar v_F^2$. Solving the system (5.1), (5.2) and using the formula (1.4), one obtains the following expressions for densities of current:

$$j_x^{(n)} = -en_e u_x^{(e)} = \sigma^{(n)} E_{1x} = \frac{e}{2\pi\beta}\left(\frac{eV_1}{v_F\hbar}\right)^2 D^{(n)} E_{1x}, \qquad (5.4)$$

$$j_x^{(p)} = en_h u_x^{(h)} = \sigma^{(p)} E_{1x} = \frac{e}{2\pi\beta}\left(\frac{eV_2}{v_F\hbar}\right)^2 D^{(p)} E_{2x}. \qquad (5.5)$$

Here, $D^{(n)}$ and $D^{(p)}$ are the drag factors, responsible for electron-hole drag in the considered bilayer graphene structure. They are

$$D^{(n)} = \frac{(\beta/\xi) - [(E_{2x}/E_{1x}) - (V_1/V_2)^2]}{(\beta/\xi) + [1 + (V_1/V_2)^2]},$$

$$D^{(p)} = \frac{(\beta/\xi) - [1 - (V_1/V_2)^2 (E_{1x}/E_{2x})]}{(\beta/\xi) + [1 + (V_1/V_2)^2]}. \qquad (5.6)$$

In absence of drag ($\xi \to 0$), we have $D^{(n)} = D^{(p)} = 1$. The symmetry of conductivities $\sigma^{(n)}$ and $\sigma^{(p)}$ relatively permutation $1 \leftrightarrow 2$ is evident. So, it is sufficient to investigate the structure of one drag factor.

Below Figs. 2–4 illustrate dependence of the drag factor $D^{(n)}$ on the value of three controlling combinations. If the factor $D^{(n)}$ changes its sign, it means that drag of electrons in *n*-layer by drifting holes in *p*-layer is forced to reverse the direction of electron current.

6. Galvanomagnetic effects in grapheme

Galvanomagnetic effects take up very important place in kinetics of solid state systems. Measurements of conductivity give data, which have to be supplemented by additional results. Really, the conductivity contains at least two ingredients: carrier density and relaxation parameter (mobility). Therefore, some need appears to execute in practice a complex set of experimental measurements. Galvanomagnetic effects are quite suitable for this purpose. Below, we will consider magnetoresistance and Hall effect.

6.1. Neutral graphene

Here using Eqs. (2.12), one obtains the following equations from the balance ones ($\beta > 0$; $\xi > 0$):

$$-\left[\vec{E}_\perp + (1/c)\left(\vec{H} \times \vec{u}_\perp^{(e)}\right)\right] - \beta \vec{u}_\perp^{(e)} - \xi\left(\vec{u}_\perp^{(e)} - \vec{u}_\perp^{(h)}\right) = 0,$$

$$\left[\vec{E}_\perp + (1/c)\left(\vec{H} \times \vec{u}_\perp^{(h)}\right)\right] - \beta \vec{u}_\perp^{(h)} - \xi\left(\vec{u}_\perp^{(h)} - \vec{u}_\perp^{(e)}\right) = 0. \qquad (6.1)$$

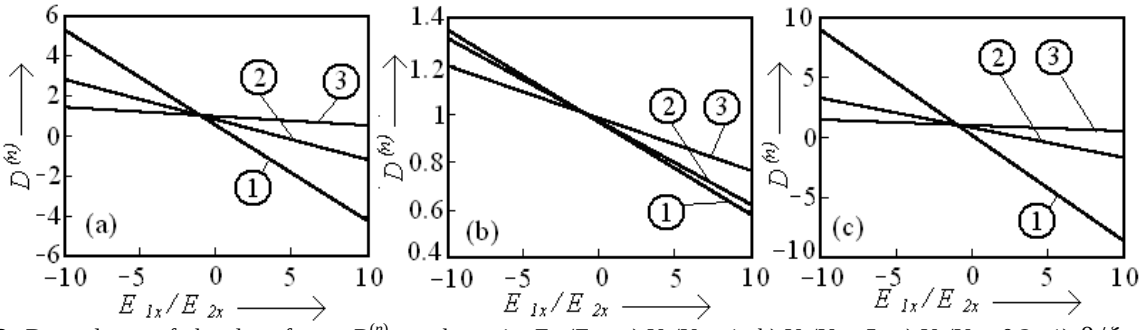

Fig. 2. Dependence of the drag factor $D^{(n)}$ on the ratio E_{1x}/E_{2x}. a) $V_1/V_2 = 1$, b) $V_1/V_2 = 5$, c) $V_1/V_2 = 0.2$; 1) $\beta/\xi = 0.1$, 2) $\beta/\xi = 3$, 3) $\beta/\xi = 20$.

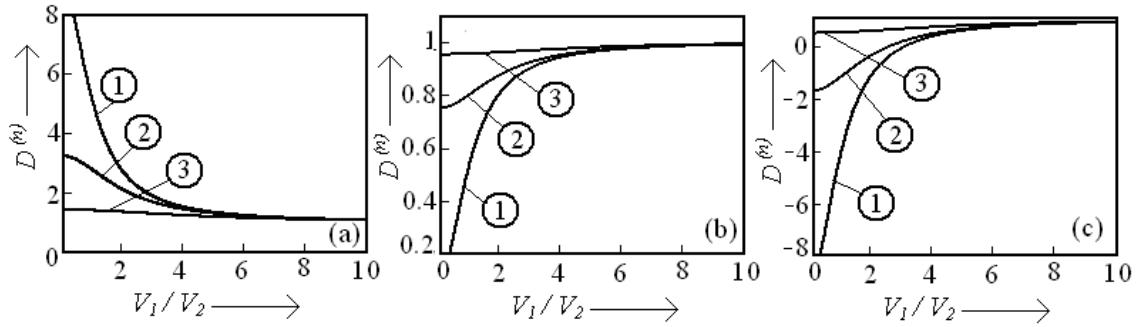

Fig. 3. Dependence of the drag factor $D^{(n)}$ on the ratio V_1/V_2. a) $E_{1x}/E_{2x} = -10$, b) $E_{1x}/E_{2x} = 0$, c) $E_{1x}/E_{2x} = 10$; 1) $\beta/\xi = 0.1$, 2) $\beta/\xi = 3$, 3) $\beta/\xi = 20$.

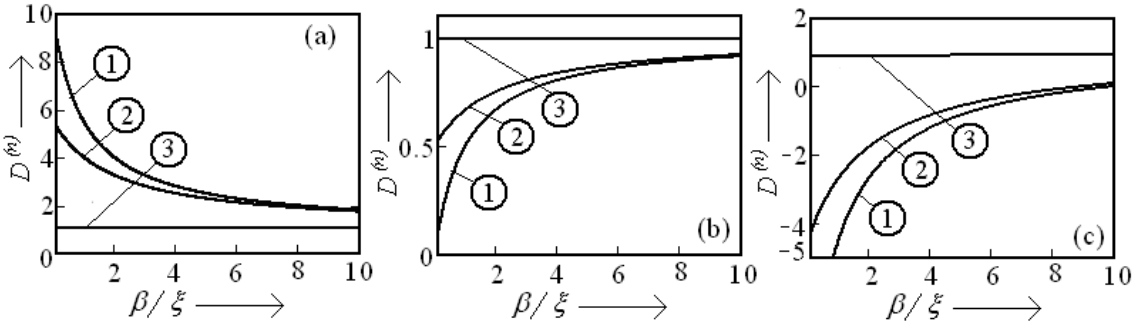

Fig. 4. Dependence of the drag factor $D^{(n)}$ on the ratio β/ξ. a) $E_{1x}/E_{2x} = -10$, b) $E_{1x}/E_{2x} = 0$, c) $E_{1x}/E_{2x} = 10$; 1) $V_1/V_2 = 0.1$, 2) $V_1/V_2 \to 0$, 3) $V_1/V_2 = 10$.

Solution of this system is as follows:

$$\vec{u}_\perp^{(e)} = [(1/bc)(\vec{H}\times\vec{E}_\perp) - \vec{E}_\perp]/(H_z^2/c^2\beta + \beta + 2\xi),$$

$$\vec{u}_\perp^{(h)} = [(1/bc)(\vec{H}\times\vec{E}_\perp) + \vec{E}_\perp]/(H_z^2/c^2\beta + \beta + 2\xi). \quad (6.2)$$

Then, the density of current is

$$\vec{j}_\perp = \vec{j}_\perp^{(h)} + \vec{j}_\perp^{(e)} = en^{(e)}(\vec{u}_\perp^{(h)} - \vec{u}_\perp^{(e)}) = \\ = [2en^{(e)}(H_z^2/c^2\beta + \beta + 2\xi)^{-1}]\vec{E}_\perp. \quad (6.3)$$

It follows from Eq. (6.3) that in neutral graphene $\vec{j}_\perp \| \vec{E}_\perp$; that is $\sigma_{xy} = 0$, and the Hall field normal to the total current does not arrive. What concerns transverse magnetoresistivity, it exists here:

$$\sigma(H_z) = 2en^{(e)}/(H_z^2/c^2\beta + \beta + 2\xi),$$
$$\sigma(0) = 2en^{(e)}/(\beta + 2\xi), \quad (6.4)$$

These two formulae show the possibility to find two kinetic coefficients β and ξ by performing not complicated experimental measurements. Note that for high magnetic fields the dependence of conductivity on the drag coefficient ξ disappears.

6.2. Gated graphene

Consider here *n*-graphene created by applying the sufficiently high voltage V_e to a gate ($eV_e >> k_BT$). Then, the balance equation for electrons has the form (see Eq. (6.1))

$$\left[\vec{E}_\perp + (1/c)\left(\vec{H}\times\vec{u}^{(e)}\right)\right] + \beta\,\vec{u}_\perp^{(e)} = 0. \quad (6.5)$$

The density of current $\vec{j}_\perp = -en_e\vec{u}_\perp^{(e)}$.

Solving the vector equation (3.2), one obtains:

$$u_x^{(e)} = -[\beta E_x + (H_z/c)E_y]/[\beta^2 + (H_z/c)^2],$$
$$u_y^{(e)} = [-\beta E_y + (H_z/c)E_x]/[\beta^2 + (H_z/c)^2]. \quad (6.6)$$

Let the current is directed along the *x*-axis: $\vec{j}_\perp = (j_x, 0)$. Determine the Hall constant R_H and magnetic conductivity $\sigma(H)$ by using the relations

$$E_y = R_H H_z j_x, \quad j_x = \sigma(H)E_x. \quad (6.7)$$

It follows from Eqs. (6.6) that for an arbitrary magnetic field

$$\sigma(H) = en^{(e)}/\beta, \quad R_H = (en^{(e)}c)^{-1}. \quad (6.8)$$

Thereof, one can see that the effect of transverse magnetoconductivity of exactly monopolar graphene is not pronounced (the conductivity does not depend on the magnetic field). The Hall constant does not contain any values responsible for the rate of momentum relaxation.

Note that the formulae (6.8) are distinct as compared to well-known formulae obtained using the standard τ-approximation. In particular, the deduced here Hall constant R_H does not involve into its structure the so-called "Hall factor", which depends on some specific time of relaxation. Appearance of the latter factor in literature is totally related with the principal imperfection of the method for τ-approximation (see, for instance, Ref. [9]).

Our consideration was grounded on a small number of assumptions, and they were not too hard. But one reason (see Section 7 below) is sufficiently important.

7. Galvanomagnetic effects in space limited crystals

For the magnetic field normal to the applied electric field, the ordered motion of carriers becomes two-dimensional. The value and distribution of transverse flows depend significantly on geometry of crystal. For a sufficiently long crystal, galvanomagnetic phenomena are rather simple. But the situation changes essentially when transverse sizes of crystal become comparable with the longitudinal ones.

Up to this point, our consideration has concerned crystals, the size of which l_x along the direction of total two-dimensional current was so long that the role of transverse sizes l_y appeared totally unimportant. Practically, this approach contains the definite assumption that is not valid for a general case. Here, in this Section, one considers that the case when relation of crystal width l_y to its length l_x is not extremely small and investigates the influence of the value of the ratio $L = l_x/l_y$ on galvanomagnetic effects.

Consider crystal with electron conductivity. Assume that charged carriers move in good order in *x*- and *y*-directions. In *z*-direction the crystal is not limited, and in *xy*-plane it covers the rectangular area

$$-l_x/2 \le x \le l_x/2, \quad -l_y/2 \le y \le l_y/2. \tag{7.1}$$

The external magnetic field is directed along the *z*-axis. Then vectors of the electric field $\vec{E}$ and of current density $\vec{j}$ lay in *xy*-plane:

$$\vec{j} = \left(j_x(x,y); j_y(x,y); 0\right),$$

$$\vec{E} = -\vec{\nabla}\varphi(x,y) = \left(-\frac{\partial\varphi(x,y)}{\partial x}; -\frac{\partial\varphi(x,y)}{\partial y}; 0\right) = \tag{7.2}$$

$$= (E_x; E_y; 0).$$

The current contacts are considered as disposed on the lines $x = \pm l_x/2,\ -l_y/2 \le y \le l_y/2$ and Hall contacts are located in points $x = \pm l_x/2;\quad y = \pm l_y/2$.

In accordance with (7.2) and (7.3), the components of vector $\vec{j}$ are:

$$j_x = \sigma_{xx}E_x + \sigma_{xy}E_y = -\sigma_{xx}\frac{\partial\varphi}{\partial x} - \sigma_{xy}\frac{\partial\varphi}{\partial y};$$

$$j_y = \sigma_{yx}E_x + \sigma_{yy}E_y = -\sigma_{xy}E_x + \sigma_{xx}E_y = \sigma_{xy}\frac{\partial\varphi}{\partial x} - \sigma_{xx}\frac{\partial\varphi}{\partial y}. \tag{7.3}$$

Here,

$$\sigma_{xx}(h) = \sigma_{yy}(h) = \frac{en}{\beta(1+h^2)}, \tag{7.4}$$

$$\sigma_{xy}(h) = -\sigma_{yx}(h) = \frac{en\,h}{\beta(1+h^2)} = h\sigma_{xx}(h);$$

$$h = -H_z/\beta c. \tag{7.5}$$

In the stationary case:

$$\operatorname{div}\vec{j} = 0; \quad \text{or} \quad \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} = 0. \tag{7.6}$$

Introducing (7.3) into (7.6), one obtains the following equation for the electrostatic potential $\varphi(x,y)$ (two-dimensional Laplas equation):

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0. \tag{7.7}$$

The boundary conditions for this problem are

$$\varphi(x = l_x/2, y) = \varphi_0/2;$$
$$\varphi(x = -l_x/2, y) = -\varphi_0/2; \tag{7.8}$$

$$j_y(x, y = \pm l_y/2) = \left(\sigma_{xy}\frac{\partial\varphi}{\partial x} - \sigma_{xx}\frac{\partial\varphi}{\partial y}\right)_{y=\pm l_y/2} = 0,$$

$$\text{or} \quad \left(h\frac{\partial\varphi}{\partial x} - \frac{\partial\varphi}{\partial y}\right)_{y=\pm l_y/2} = 0. \tag{7.9}$$

The average density of current is

$$j = \langle j_x \rangle = \frac{1}{l_y}\int_{-l_y/2}^{l_y/2} j_x(x,y)dy. \tag{7.10}$$

In accord with the equations (7.6) and (7.7), this value does not depend on the *x*-coordinate. Substituting (7.3) into (7.10), we obtain:

$$\langle j_x \rangle = -\frac{1}{l_y}\int_{-l_y/2}^{l_y/2}\left(\sigma_{xx}\frac{\partial\varphi(x,y)}{\partial x} + \sigma_{xy}\frac{\partial\varphi(x,y)}{\partial y}\right)dy =$$

$$= -\frac{en}{\beta(1+h^2)l_y}\times$$

$$\times\left[\frac{\partial}{\partial x}\int_{-l_y/2}^{l_y/2}\varphi(x,y)dy + h\left[\varphi(x,l_y/2) - \varphi(x,-l_y/2)\right]\right]. \tag{7.11}$$

The experimentally measured longitudinal conductivity $\sigma(H)$ is now determined by the expression

$$\sigma\ (H) = \langle j_x \rangle\, l_x/\varphi_0. \tag{7.12}$$

The value

$$V_H = \varphi^{(+)} - \varphi^{(-)}, \tag{7.13}$$

where $\varphi^{(\pm)} = \varphi(x = 0,\ y = \pm l_y/2)$ is the measured Hall potential for the case of point contacts.

Further, we shell use the dimensionless values:

$$\xi = 2x/l_x, \quad \eta = 2y/l_x, \quad -1 \le \xi \le 1,$$
$$l_x/l_y = L, \quad -1/L \le \eta \le 1/L,$$

$$\omega(\xi,\eta) = \varphi(x,y)/\varphi_0, \quad \omega^{(\pm)} = \varphi^{(\pm)}/\varphi_0. \tag{7.14}$$

Then

$$\sigma(H) = \frac{enL}{\beta(1+h^2)}\times$$

$$\times\left[\frac{\partial}{\partial\xi}\int_{-1/L}^{1/L}\omega(\xi,\eta)\,d\eta + h\left[\omega(\xi, 1/L) - \omega(\xi, -1/L)\right]\right]_{\xi=0}. \tag{7.15}$$

The Hall constant R_H is determined using the following relation:

$$R_H = \left|\left[\varphi^{(+)} - \varphi^{(-)}\right] l_x/l_y\ H_z\ \varphi_0\ \sigma(H)\right| =$$
$$= \left|\left[\omega^{(+)} - \omega^{(-)}\right] l_x/l_y\ H_z\ \ \sigma(H)\right|. \tag{7.16}$$

The problem of calculation of space distribution for dimensionless potential $\omega(\xi,\eta)$ is related with the equation

$$\frac{\partial^2\omega}{\partial\xi^2} + \frac{\partial^2\omega}{\partial\eta^2} = 0 \quad (-1 \le \xi \le 1, \quad -1/L \le \eta \le 1/L\ ), \tag{7.17}$$

where boundary conditions have the form

$$\omega(\xi = 1, \eta) = 1/2, \quad \omega(\xi = -1, \eta) = -1/2, \tag{7.18}$$

$$\left[-h\left(\frac{\partial \omega(\xi,\eta)}{\partial \xi} \right) + \frac{\partial \omega(\xi,\eta)}{\partial \eta} \right]_{\eta = \pm 1/L} = 0 . \qquad (7.19)$$

Introduce the new variable $\Omega(\xi,\eta)$:

$$\omega(\xi,\eta) = (\xi/2) + h\,\Omega(\xi,\eta) . \qquad (7.20)$$

Then, for the function $\Omega(\xi,\eta)$ the considered problem has the following form:

$$\frac{\partial^2 \Omega(\xi,\eta)}{\partial \xi^2} + \frac{\partial^2 \Omega(\xi,\eta)}{\partial \eta^2} = 0$$

$$(-1 \le \xi \le 1; \quad -1/L \le \eta \le 1/L). \qquad (7.21)$$

The boundary conditions are:

$$\Omega(\xi = 1, \eta) = 0; \quad \Omega(\xi = -1, \eta) = 0; \qquad (7.22)$$

$$\left[-h\left(\frac{\partial \Omega(\xi,\eta)}{\partial \xi} \right) + \frac{\partial \Omega(\xi,\eta)}{\partial \eta} \right]_{\eta = \pm 1/L} = \frac{1}{2} . \qquad (7.23)$$

Solution of the problem (7.21)-(7.23) requires to perform sufficiently cumbersome calculations for given values *h* and *L* (see [11]). Usually obtained solution has a form of unlimited series, which requires an approximate summing with definite error. Due to some reasons especially high requirements to precision of calculations appears unnecessary. Then, it has a sense to use more simple approximate methods of calculations. Seeking for the suitable form of the function $\Omega(\xi,\eta)$, assume the following symmetry:

$$\Omega(\xi, \eta) = -\Omega(\xi, -\eta); \; \Omega(\xi, \eta) = -\Omega(-\xi, \eta) . \qquad (7.24)$$

Supposing the value $\Omega(\xi,\eta)$ even for parameter *h*, in future for the convenience one accepts temporarily $h \ge 0$. In accordance with (7.24), it is sufficient to consider the area

$$0 \le \xi \le 1; \quad 0 \le \eta \le 1/L . \qquad (7.25)$$

The approximate solution of the problem (7.21)-(7.23) is performed by us using the following model function:

$$\Omega(\xi, \eta) = (\eta/2)\,\lambda(\xi). \qquad (7.26)$$

Substituting the form (7.26) into (7.23), one obtains the equation

$$-\frac{h}{L}\frac{d\lambda(\xi)}{d\xi} + \lambda(\xi) = 1. \qquad (7.27)$$

In agreement with (7.22)

$$\lambda(\xi = 1) = 0. \qquad (7.28)$$

Solution of the problem (7.27), (7.28) is

$$\lambda(\xi) = 1 - \exp\left[-L(1-\xi)/h\right]. \qquad (7.29)$$

Thereof, it follows (see (7.26)):

$$\Omega(\xi, \eta) = (\eta/2)\left\{1 - \exp\left[-L(1-\xi)/h\right]\right\}. \qquad (7.30)$$

Using the formulae (7.20), (7.24), and (7.30), one obtains for an arbitrary sign of *h* the approximate solution for the dimensionless potential over the whole area ($-1 \le \xi \le 1$ and $-1/L \le \eta \le 1/L$):

$$\omega(\xi,\eta) = (\xi/2) + (h\eta/2)\left\{1 - \exp\left[\left(|\xi| - 1\right)L/|h|\right]\right\}. \qquad (7.31)$$

Note also (see (7.14) and (7.20)):

$$\omega^{(+)} = -\omega^{(-)} = (h/2L)[1 - \exp(-L/|h|)] . \qquad (7.32)$$

The function (7.31) is not a precise solution of the problem (7.17)-(7.19). It satisfies the corresponding boundary conditions but does not satisfy precisely the Laplas equation (7.17). But at the condition $L >> |h|$, the structure $[\partial^2\omega(\xi,\eta)/\partial\xi^2] + [\partial^2\omega(\xi,\eta)/\partial\eta^2]$ with the form (7.31) practically in the whole area is exponentially small value. The limit transition $L \to \infty$ gives well known result. Note also that, at the condition $L << |h|$, the function (7.31) is also an approximate solution of a good manner for the Laplas equation.

It follows from the form (7.31) that distribution along *y*-axis of electric potential, dependent on magnetic field, is determined by the parameter *h*, along *x*-axis – by the parameter L/h.

Introducing the formulae (7.31) and (7.32) into the expressions (7.15) and (7.16), one obtains the following forms:

$$\sigma(H) = (en/\beta)\cdot G_{(\sigma)}(h; L) ,$$

$$R_H = (enc)^{-1}\cdot G_{(R)}(h; L) , \qquad (7.33)$$

where

$$G_{(\sigma)}(h, L) = \frac{1}{1+h^2}\left\{1 + h^2\left(1 - \exp(-L/h)\right)\right\}; \qquad (7.34)$$

$$G_{(R)}(h\,, L) = \frac{(1+h^2)\left[1 - \exp(-L/h)\right]}{1 + h^2\left(1 - \exp(-L/h)\right)} . \qquad (7.35)$$

Note that $G_{(R)}(h\,; L \to \infty) \to 1$ and $G_{(\sigma)}(h\,; L \to \infty) \to 1$. The functions (7.34) and (7.35) depend on two arguments: dimensionless magnetic field *h* and ratio of two geometrical sizes *L*. The factors $G_{(\sigma)}(h\,; L)$ and $G_{(R)}(h\,; L)$ have to be considered as geometrical factors for "longitudinal" magnetoconductivity and "Hall constant".

The geometrical factor of magnetoresistance $G_{(\rho)}(h, L)$ is introduced as

$$G_{(\rho)}(h, L) = 1/G_{(\sigma)}(h, L) . \qquad (7.36)$$

Note that to the relation

$$r(h, L) = R_H(h, L)/R_H(\infty, L) = \\ = G_{(R)}(h, L)/G_{(R)}(\infty, L) . \qquad (7.37)$$

one usually gives the title Hall factor. It follows from Eqs. (7.33)-(7.35) that the influence of magnetic field on longitudinal conductivity and Hall constant of monopolar crystal is related with a limited ratio of sizes of the crystal.

It is seen from Figs. 5 and 6 (the calculations were performed using the formulae (7.34) and (7.35)) that longitudinal conductivity and Hall coefficient in extremely long specimen ($l_x/l_y = L \to \infty$) do not depend on the intensity of magnetic field. If the specimen is short, longitudinal conductivity and Hall coefficient decrease with increasing the magnetic field. At very high magnetic fields, the Hall coefficient tends

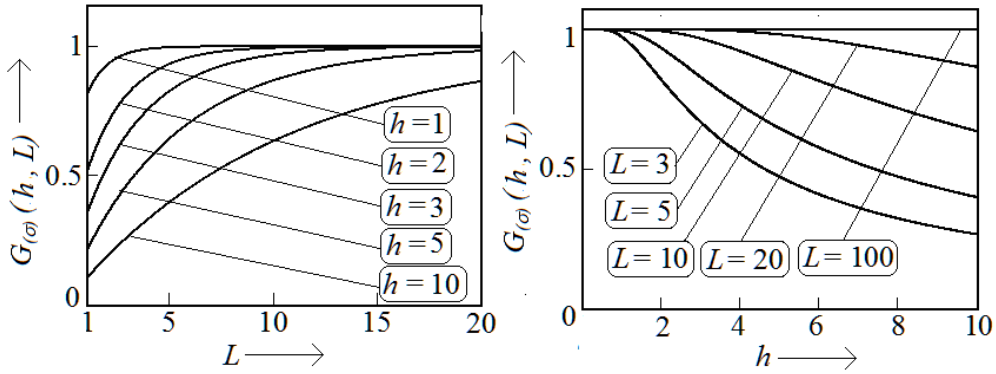

Fig. 5. Dependence of the geometrical factor for magnetoconductivity on the ratio of geometrical sizes of crystal (L) and on the value of magnetic field (h).

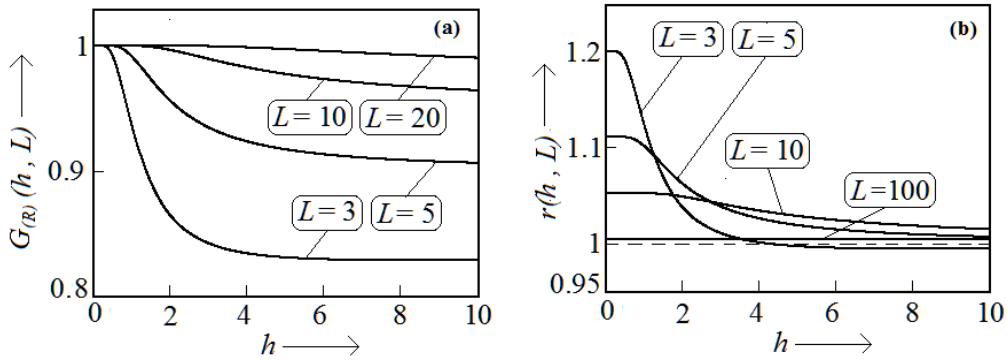

Fig. 6. Dependence of the geometrical factor for the Hall constant (a) and Hall factor (b) on the value of magnetic field for various ratios of crystal geometrical sizes.

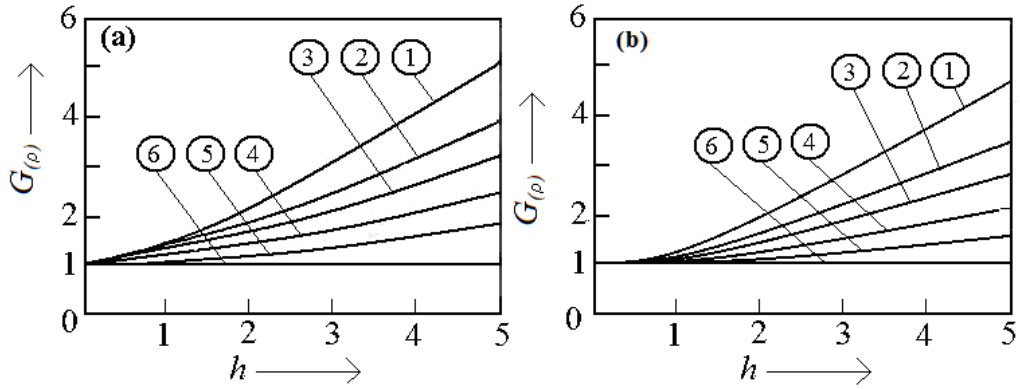

Fig. 7. Comparison of exact (a) and approximate (b) dependences for the geometrical factor of magnetoconductivity on the value of magnetic field. 1) L = 1, 2) L = 1.5, 3) L = 2, 4) L = 3, 5) L = 5, 6) L = ∞.

to constant value that depends only on the value of ratio L. Magnetoconductivity does not approach saturation and sufficiently decreases when magnetic field rises.

The Figs 7a and 8a show geometrical factors for magnetoconductivity and the Hall coefficient precisely calculated for the problem (7.7)-(7.9) by totally numerical way (see Ref. [12]). Figs 7b and 8b show the same patterns calculated using the model potential (7.31). It is seen that distinction between figures (a) and (b) is not so large (especially for high L). The good agreement of results obtained by exact and approximate ways is not occasional. The used approximate form of electrical potential provides continuity of current at all the boundaries of crystal, where that is really necessary.

Presented here Figs. 7 and 8 show that in short crystals magnetic field influences impressivebly on conductivity and the Hall constant of crystal.

8. Discussion

It was shown above that in unlimited neutral graphene crystal Hall effect is absent, while magnetoresistance is present; in unlimited monopolar graphene the situation is

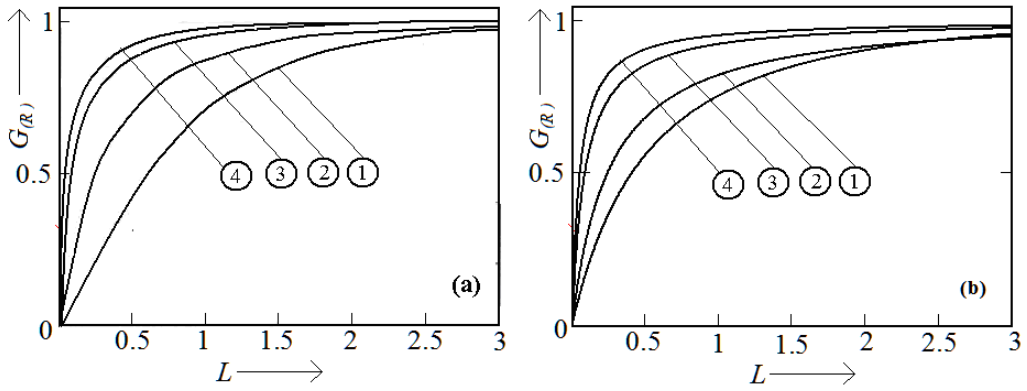

Fig. 8. Comparison of exact (a) and approximate (b) dependences for the geometrical factor of the Hall constant on the ratio of crystal geometrical sizes.
1) h = 1.428, 2) h = 3.732, 3) h = 11.43, 4) h = 19.08.

totally opposite: Hall effect is present, but magnetoresistance is absent. In a space-limited crystal (ratio length to width is not too high), Hall effect and magnetoresistance are present both in neutral and monopolar crystals.

Appendix

As external scattering system, we consider here impurities distributed uniformly in plane $z = 0$ with the density $n_I^{(2)}$.

A1. "Neutral" hydrogen-like impurities with the density $n_{NI}^{(2)}$

Three-dimensional potential of individual impurity is (see Ref. [9])

$$\varphi_{NI}(\vec{r}) = (e_I / \varepsilon_L r)(1 + rq_B / 2)\exp(-rq_B);$$

$$q_B = 2m_0 e^2 / \varepsilon_L \hbar^2 .$$

After Fourier transformation, one obtains:

$$\varphi_{NI}(\vec{q},\omega) = [4\pi e_I / \varepsilon_L (q^2 + q_B^2)]\,[1 + q_B^2/(q^2 + q_B^2)]\,\delta(\omega),$$

$$\varphi_{NI}(\vec{q}_\perp,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\varphi_{NI}(\vec{q},\omega)\,dq_z =$$

$$= \frac{2\pi e_I}{\varepsilon_L\sqrt{q_\perp^2 + q_B^2}}\left[1 + \frac{q_B^2/2}{q_\perp^2 + q_B^2}\right]\delta(\omega).$$

For $q_B >> q_\perp$ (see also Ref. [6]),

$$\langle \varphi_{NI}^2 \rangle_{q_\perp} = 9\pi e^2 n_{NI}^{(2)} / 2\varepsilon_L^2 q_B^2 . \qquad (A.1)$$

A2. Charged impurities with density $n_{CI}^{(2)}$

In this case (see Ref. [6]),

$$\varphi_{CI}(\vec{r}) = \frac{e_I}{\varepsilon_L r}; \quad \varphi_{CI}(\omega,\vec{q}) = \frac{4\pi e_I}{\varepsilon_L q^2}\delta(\omega);$$

$$\langle \varphi_{CI}^2 \rangle_{\vec{q}_\perp} = \frac{1}{2\pi}\int \langle \delta\varphi_{CI}^2 \rangle_{\vec{q}}\; dq_z = \frac{(2\pi)^3 e^2 n_{CI}^{(2)}}{\varepsilon_L^2\, q_\perp^2}. \quad (A.2)$$

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