Semiconductor Physics, Quantum Electronics & Optoelectronics, 2020. V. 23, No 2. P. 136-140.

Semiconductor physics

Rotation of a thin heated plate caused by its own coherent thermal radiation

V.I. Pipa, A.I. Liptuga

V. Lashkaryov Institute of Semiconductor Physics, NAS of Ukraine,
41, prospect Nauky, 03680 Kyiv, Ukraine
E-mail: lipant@ukr.net

Abstract. Presented in this paper are the results of theoretical studying the rotational motion of a heated solid-state plate under action of its coherent asymmetric thermal radiation. Thin plane-parallel plates are considered as objects, in which the coherence of thermal radiation is due to the interference effects. Time dependences of the plate rotation angle have been calculated under conditions that the plate temperature is kept constant as well as in the mode of its radiation cooling. It has been shown that the coherence of thermal radiation can lead to that the rotation and cooling rates of a plate with a certain thickness can exceed the corresponding values not only for thicker plates, but for thinner ones as well. The numerical calculations have been performed for crystalline plates with the temperature 350 K and the ambient one 300 K.

Keywords: coherent thermal radiation, photon recoil, rotation of a thin plate, radiative cooling of the plate.

https://doi.org/10.15407/spqeo23.02.136
PACS 06.30.Gv, 44.40.+a

Manuscript received 19.02.20; revised version received 24.03.20; accepted for publication 10.06.20; published online 12.06.20.

1. Introduction

As a result of photon recoil, the force in the direction opposite to the thermal radiation (TR) flow acts on a heated body that emits TR. If the TR emission is asymmetric, then moments of forces can also act on the heated body. Thus, the heated free body can perform both translational and rotational motion. The photon recoil effects play an important role for both the atoms (the Mössbauer effect) and nanotechnological objects [1, 2], and for cosmic bodies (rotation of asteroids [3], anomaly in the acceleration of "Pioneer 10" and "Pioneer 11" spacecrafts [4, 5]). The methods for numerical calculation of photon recoil for heated bodies of arbitrary shapes were developed in [6]. Specifically, in [6], the motion of heated Janus and Rytov microparticles was studied, which being heated begin to accelerate in a certain direction or rotate, respectively.

The rotational motion of a uniformly heated thin plate under the action of its incoherent TR was theoretically studied in [7]. The calculations were performed for a plate which thickness is large as compared to the wavelength at the maximum of the black body light spectrum corresponding to the plate temperature. This report is devoted to studying the above effect for optically thin plane-parallel plates, which TR has a pronounced temporal and spatial coherence due to multibeam interference [8–11].

2. Results and discussion

Let us consider a thin rectangular plate with uniform distribution of temperature T; the ambient temperature is T_1. The pressure on the plate is exerted by both its own TR and background radiation. Assuming that the lengths l_x and l_z of the edges of wide plate faces significantly exceed the plate thickness l_y and the characteristic wavelengths of the plate TR and background radiation, we take into account the pressure only on the wide faces and neglect the edge effects in the radiation and reflection of the waves.

The resulting pressure P (difference between the values of pressure on the plate opposite faces) is determined by the expression [7]:

$$P = c^{-1}\iint (R_1 - R_2)(J_0(T) - J_0(T_1))d\lambda d\Omega\,. \qquad (1)$$

Here, $R_j(\lambda,\vartheta)$ is the reflection coefficient of unpolarized radiation with a wavelength λ incident from vacuum onto the plate surfaces $j = 1, 2$ in the solid angle element $d\Omega$ at an angle ϑ to their normals; $J_0(T) = 4\pi\hbar c^2\lambda^{-5}N(T)\cos\vartheta$; $N(T)$ is the Planck function. Integration over the solid angle is carried out in the hemisphere. If $R_1 > R_2$ and $T > T_1$, then the pressure P (Eq. (1)) acts externally on the surface with a lower reflection coefficient.

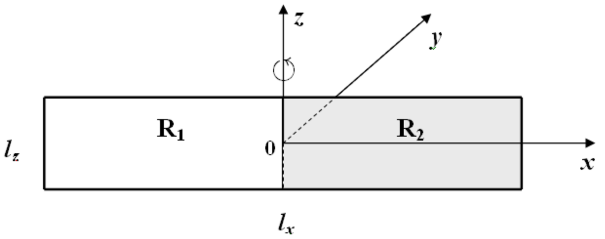


Fig. 1. Diagram of the wide front face of a thin plate (thickness) $l_y << l_x$, l_z. The light and darkened halves of the face differ in the reflection coefficients of radiation incident from vacuum on these parts of the plate ($R_1 \neq R_2$). The location of these sections on the back face is obtained from the presented location for the front face, when the plate rotates by 180° about z-axis.

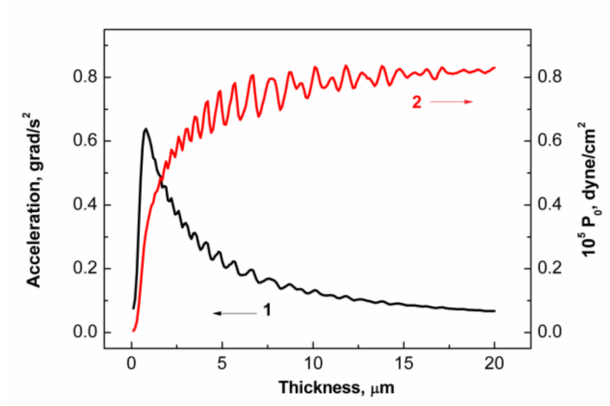


Fig. 2. Thickness dependence of angular acceleration a (*1*) and pressure P_0 (*2*) for a germanium plate with the temperature $T = 350$ K and the background temperature $T_1 = 300$ K (plate length $l_x = 2$ cm, density $\rho = 5.32$ g/cm^3). The coefficients $R_1 = 1$ and $R_2 = R$ determine the intensities of the radiation reflected from the plate, incident from the vacuum onto a metallized and polished surface, respectively.

To study the effect of radiation rotation of the heated plate, we use the model considered in [7]. Shown in Fig. 1 is the wide front face of the plate, on the left and right halves of which the reflection coefficients for radiation from the plate are equal to R_1 and R_2, respectively. The same areas on the back face of the plate are opposite to this, so that z-axis drawn through the center of mass is the axis of symmetry C_2. In such a structure, at $R_1 \neq R_2$ the TR pressure creates rotational moments of a pair of forces and causes rotation of the free plate around z-axis. The direction of rotation (shown by a circular arrow) corresponds to the rotation of the heated plate with the coefficients $R_1 > R_2$.

It should be noted that, for the model under consideration, gravity does not contribute to the moment of forces relatively to z-axis. So, this axis can be oriented both vertically and horizontally.

Let us perform calculations for a plane-parallel crystalline plate with the reflection coefficients $R_1 = 1$ and $R_2 = R(\lambda, \vartheta, l_y)$. The coefficient R corresponds to the reflection of unpolarized radiation incident from vacuum onto the polished surface of the plate, back surface of which borders with an opaque metal layer. We use semiconductors for which the spectral maximum of the equilibrium radiation of a black body within the temperature range from T_1 to T falls into the spectral region of absorption by free electrons. In this case, the dielectric function at a frequency $\omega = 2\pi c/\lambda$ is

$$\varepsilon = \varepsilon_\infty + i\frac{\omega_p{}^2\tau}{\omega(1 - i\omega\tau)}, \quad \omega_p^2 = \frac{4\pi e^2 N_e}{m^*}. \tag{2}$$

Here, ε_∞ is the high-frequency permittivity, ω_p – plasma frequency, N_e, m^* and τ are the concentration, effective mass, and electron relaxation time, respectively. For numerical calculations, we use the n-Ge parameters [12]: $\varepsilon_\infty = 16$, $N_e = 10^{19}$ cm^{-3}, $\tau = 2.5\times10^{-14}$ s, and $m^* = 0.16m_0$ (m_0 is the mass of free electron).

Let the plate becomes free and begin to rotate at the point of time $t = 0$. The azimuthal angle $\varphi(t)$, by which the plate rotates in time t, is defined by the equation

$$I_z \frac{d^2\varphi}{dt^2} = K \tag{3}$$

with initial conditions $\varphi(0) = 0$, $\dot{\varphi}(0) = 0$. Here, I_z is the moment of inertia of the plate and $K = Pl_x{}^2 l_z/4$ is the moment of a pair of pressure forces P with respect to z-axis (the action of other forces is neglected). In approximation $(l_y/l_x)^2 << 1$, $I_z = ml_x{}^2/12$, where m is the plate mass.

We will study the kinetics of plate rotation under the same temperature conditions as in [7]. It is assumed that at $t > 0$: 1) the temperature of heated plate is kept constant, and 2) heating of the plate stops, and it is cooled by its own TR to the background temperature T_1.

2.1. Constant temperature

Under the action of constant pressure $P_0 = P(T_0, T_1)$, the plate with a constant temperature $T(t = 0) = T_0$ begins to rotate uniformly accelerated, $\varphi = at^2/2$, with the angular acceleration

$$a = \frac{3P_0 l_z}{m}. \tag{4}$$

Shown in Fig. 2 are the dependences of acceleration a and pressure P_0 on the plate thickness l_y, calculated neglecting the thickness of the metal coating, which provides a mirror reflection of radiation ($R_1 = 1$).

The sharp peak of the function $a(l_y)$ corresponds to the thickness $l_y = 0.8$ μm $= l^{opt}$. The oscillatory nature of the $P_0(l_y)$ and $a(l_y)$ dependences is caused by the contribution of multipath interference to the reflection coefficient R. The type of function $a(l_y)$ is defined by two competing factors: the dependences of pressure P_0 and plate mass on the thickness l_y. As the plate thickness decreases, the function $P_0(l_y)$ averaged over the oscillations decreases. This factor decreases the acceleration, and the corresponding decrease in mass ($m \propto l_y$) increases it. If $l_y > l^{opt}$, then, in the thickness dependence of the acceleration averaged over the oscillations, the factor of mass change dominates. As a result, with an increase in the plate thickness, the averaged acceleration monotonously decreases and asymptotically approaches the acceleration of the same plate, but with incoherent TR.

In the region $l_y < l^{opt}$, the slope of the $P_0(l_y)$ curve increases significantly and with decreasing l_y pressure becomes so weak that this factor leads to a decrease in acceleration. For example, as the thickness decreases from l^{opt} to $l^{opt}/2$, the mass decreases by two times, and the pressure decreases by about four times. Such a sharp drop in pressure is explained by the peculiarities of the TR spectra and reflected background radiation due to interference.

Indeed, one can see from Fig. 3 that the spectral density of specific pressure $p(\lambda)/l_y$, where

$$p(\lambda) = 4\pi\hbar c\lambda^{-5}\int(1-R)\left(N(T_0)-N(T_1)\right)\cos\vartheta d\Omega\,, \quad (5)$$

has the form of isolated peaks for thin plates (curves *1* and *2*) and oscillations for a thicker plate (curve *3*). The acceleration $a(l_y)$ is proportional to the area under the corresponding spectral curve. The area under curve *1* is larger than those under curves *2* and *3*. Thus, under the action of coherent TR, the plate with a thickness l^{opt} rotates faster not only than the thicker plate, but also faster than the thinner plate (or, in other words, faster than the heavier and lighter ones). The nonmonotonic dependences of the angular acceleration of heated Rytov microparticles on their diameter obtained in the theory [6] are caused by the chirality of microparticles.

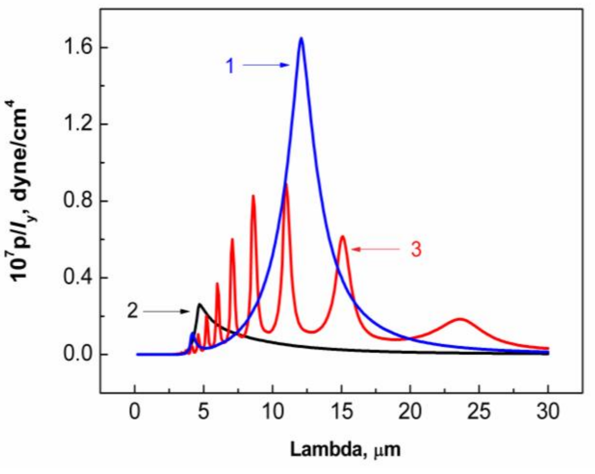


Fig. 3. Spectral density of specific pressure $p(\lambda)/l_y$. Curves *1*, *2*, and *3* refer to germanium plates with the thickness l_y (μm) = 0.8, 0.3, and 5, respectively. The reflection coefficients R_1, R_2 and temperatures T_0, T_1 are the same as for Fig. 2.

2.2. Radiative cooling effect

Radiation cooling of a heated body is determined by the difference between the arrival and departure of radiation energy. A plate with the reflection coefficients $R_1 = 1$ and $R_2 = R(\lambda, \vartheta)$ is characterized by emissivities $A_1 = 0$, $A_2 = 1 - R(\lambda, \vartheta)$. Its cooling at $t > 0$ is described by the equation [7]:

$$\rho l_y C_p \frac{dT}{dt} = -cP(T, T_1) \quad (6)$$

with the initial condition $T(0) = T_0$. Here, C_p is the specific heat, pressure P is determined by Eq. (1). Other cooling mechanisms as well as changes in semiconductor parameters with decreasing the temperature from $T_0 = 350$ K to $T_1 = 300$ K are not taken into account.

Using the initial conditions $\varphi(0) = 0$, $\dot{\varphi}(0) = 0$, we obtain from Eqs (3) and (6):

$$t = -\rho l_y C_p \int_T^{T_0} (cP)^{-1} dT\,, \quad (7)$$

$$\varphi(t) = C_r\left[(T_0 - T)t - \int_T^{T_0} t dT\right], \quad (8)$$

where $C_r = 3C_p/(cl_x)$. It follows from Eqs (7) and (8) that, at the initial stage of cooling ($T_0 - T(t) << T_0$), the temperature of the plate decreases linearly:

$$T(t) = T_0 - \frac{a}{C_r}t\,, \quad (9)$$

and the rotational motion is uniformly accelerated with the angular acceleration a defined by Eq. (4). With t growth, when $T \to T_1$, the angular velocity $\dot{\varphi}$ tends to a constant value $C_r(T_0 - T_1)$, and $\varphi(t)$ approaches to the linear asymptotic dependence

$$\varphi_{as}(t) = C_r\left[(T_0 - T_1)t - \int_{T_1}^{T_0} t dT\right]. \quad (10)$$

The presence of a peak in the dependence of angular acceleration on thickness (Fig. 2) leads to that the plate with a thickness l^{opt} can not only rotate faster, but also cool faster than a thicker or thinner plate. This feature of radiation cooling is well demonstrated by the linear law (see Eq. (9)). The results of numerical calculations of the time dependences of the temperature and the angle of rotation within the $T_1 < T(t) \le T_0$ range are presented in Figs 4 and 5. The method used to solve the system of Eqs (7) and (8) is described in the Appendix.

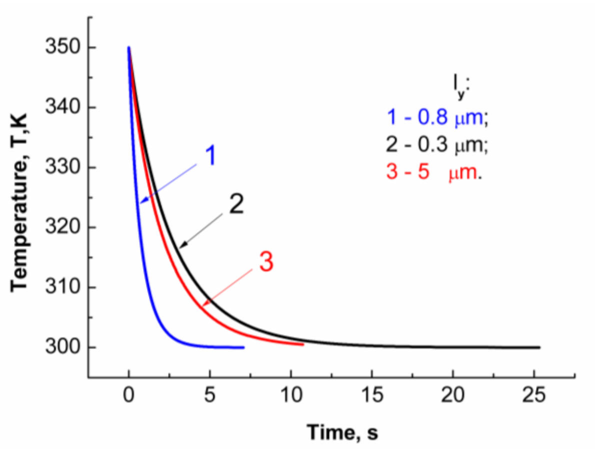


Fig. 4. Temporal temperature dependence for germanium plates with the length l_x = 2 cm and thickness l_y (μm) = 0.8, 0.3 and 5 (*1*, *2* and *3*, respectively); specific heat $C_p = 3.1\cdot10^6$ erg/(g·K).

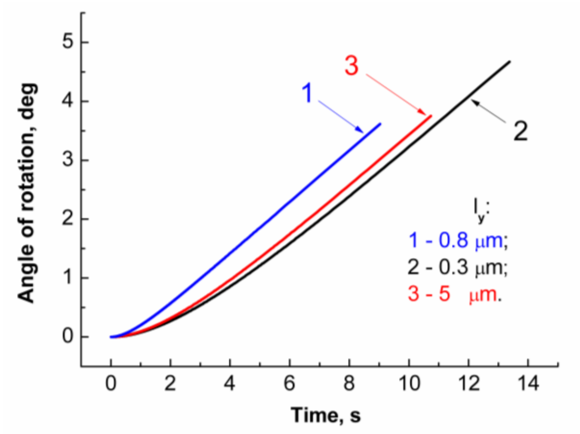


Fig. 5. Temporal dependences of the rotation angle of germanium plates, which temperature decreases over time due to their thermal radiation from the initial value T = 350 K: l_y (μm) = 0.8, 5, and 0.3 (*1*, *2*, and *3*, respectively). l_x = 2 cm.

Fig. 5 shows the dependences $\varphi(t)$ for the same plates for which Fig. 4 presents the $T(t)$ functions. One can see from Figs 4 and 5 that the most rapidly cooled plate (with the thickness l_y = 0.8 μm) also rotates by larger angle than the thicker or thinner one. The asymptotes (Eq. (10)) (linear sections of the curves in Fig. 5) correspond to the rotation of the cooled plate by inertia at a constant speed, which does not depend on the plate thickness.

3. Conclusions

This paper presents the results of a theoretical studying the rotation of a uniformly heated thin plane-parallel plate under the action of photon recoil, when asymmetrical coherent thermal radiation takes place. Radiation asymmetry and, accordingly, the rotational moment arise due to the difference in the emissivity of two halves of the wide plate faces. The linear dimensions of these faces are assumed to be large as compared to the plate thickness and the wavelength at the maximum of the black body spectrum with the plate temperature. The coherence of thermal radiation is created as a result of the resonator properties inherent to the plate.

The rotation of plate has been investigated under conditions when its initial temperature T_0 is kept constant, as well as in the mode of cooling caused by its own thermal radiation to the ambient temperature T_1. The *n*-Ge parameters and temperatures T_0 = 350 K, T_1 = 300 K were used in numerical calculations. It has been shown that, in the constant temperature mode, the plate rotates with the constant angular acceleration a, which non-trivially depends on its thickness l_y. At a certain thickness $l_y = l^{opt}$, the function $a(l_y)$ has a sharp peak, *i.e.*, the plate with a thickness l^{opt} rotates faster not only than a thicker plate, but also faster than the thinner one. The presence of a peak is explained by the spectral features of coherent thermal radiation of optically thin plates. In the regime of radiation cooling of a plate, these features form unusual temporal dependences of the plate temperature and angle of rotation: the plate with the thickness l^{opt} can not only rotate faster, but also cool faster than the thicker or thinner plate. The *n*-Ge plate with the electron concentration close to 10^{19} cm^{-3}, length 2 cm and thickness l^{opt} = 0.8 μm, at the constant temperatures T_0 = 350 K, T_1 = 300 K rotates for 60 s by the angle close to 1140° (about three full turns).

The obtained results can be of use in the development of heat engines based on photon recoil.

Appendix

Calculation of the temperature dependence on time

For a mirror-black plate (R_1 = 1, R_2 = 0), from Eqs (1) and (7), we obtain $P(T) = P_{bb}(T)$ and $t(T) = t_{bb}(T)$, where

$$P_{bb}(T) = \sigma\left(T^4 - T_1^4\right)/c\,, \quad \text{(A1)}$$

$$t_{bb}(T) = \frac{1}{2\beta T_1^3}\left(\Phi(\varsigma) - \Phi(\varsigma_0)\right),$$

$$\Phi(\varsigma) = (1/2)\ln\left(\frac{\varsigma+1}{\varsigma-1}\right) + \arctan(\varsigma)\,. \quad \text{(A2)}$$

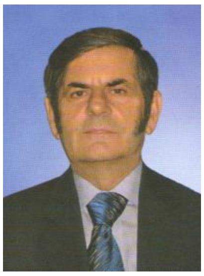
Here, σ is the Stefan–Boltzmann constant, $\varsigma = T/T_1 \geq 1$, $\varsigma_0 = T_0/T_1$ and $\beta^{-1} = \rho l_y C_p/\sigma$.

A comparison of the functions $P(T)$ and $P_{bb}(T)$ calculated within the interval $T_1 \leq T \leq T_0$ using Eqs (1) (R_1 = 1 and $R_2 \equiv R(\lambda, \vartheta)$) and (A1), respectively, shows that, with sufficient accuracy, it is possible to interpolate $P(T)$ with a function $C_{ly} P_{bb}(T)$, where the coefficient C_{ly} depends on the plate thickness. Thus, from Eq. (7) we obtain the function $t(T) = C_{ly} P_{bb}(T)$ and using it also $\varphi(t)$. The dependences presented in Figs 4 and 5 were obtained using the coefficients C_{ly} = 38.4, 4.02, 1.85 for the thickness values l_y (μm) 0.3, 0.8, 5, respectively.

References

1. Liu M., Zentgraf T., Liu Y., Bartal G., Zhang X. Light-driven nanoscale plasmonic motors. *Nature Nanotechnology*. 2010. **5**. P. 570–573. https://doi.org/10.1038/nnano.2010.128.
2. Jain V., Gieseler J., Moritz C., Dellago C., Quidant R., Novotny L. Direct measurement of photon recoil from a levitated nanoparticle. *Phys. Rev. Lett*. 2016. **116**. P. 243601. https://doi.org/10.1103/PhysRevLett.116.243601.
3. Kaasalainen M., Durech J., Warner B.D., Krugly Y.N., Gaftonyuk N.M. Acceleration of the rotation of asteroid 1862 Apollo by radiation torques. *Nature*. 2007. **446**. P. 420–422. https://doi.org/10.1038/nature05614.
4. Toth V.T., Turyshev S.G. Thermal recoil force, telemetry, and the pioneer anomaly. *Phys. Rev. D*. 2009. **79**. P. 043011. https://doi.org/10.1103/PhysRevD.79.043011.
5. Turyshev S.G., Toth V.T., Kinsella G., Lee S.C., Lok S.M., Ellis J. Support for the thermal origin of the pioneer anomaly. *Phys. Rev. Lett*. 2012. **108**, No 24. P. 241101. https://doi.org/10.1103/PhysRevLett.108.241101.
6. Reid M.T.H., Miller O.D., Polimeridis A.G., Rodriguez A.W., Tomlinson E.M., Johnson S.G. Photon torpedoes and Rytov pinwheels: Integral-equation modeling of non-equilibrium fluctuation-induced forces and torques on nanoparticles. *arXiv preprint arXiv:1708.01985* (2017).
7. Pipa V.I., Liptuga A.I. Rotation of a thin heated plate by its own coherent thermal radiation. *Ukr. J. Phys*. 2018. **63**, No 12. P. 1105–1108. https://doi.org/10.15407/ujpe63.12.1105.
8. Pekar V.S. Peculiarities of optical spectra of thin layers*. Journal of Experimental and Theoretical Physics.* 1985. **88**, No 2. P. 667–676.
9. Liptuga A.I., Shishkina N.B. Thermal radiation of semitransparent plane-parallel objects. *Infrared Physics and Technology*. 2003. **44**, No 1. P. 85–89. https://doi.org/10.1016/S1350-4495(02)00177-9.
10. Guga K.Y., Kollyukh A.G., Liptuga A.I., Morozhenko V.A., Pipa V.I. Features of thermal radiation of plane-parallel semiconductor wafers. *Semiconductors*. 2004. **38**. P. 507–511. https://doi.org/10.1134/1755879.
11. Kollyukh A.G., Liptuga A.I., Morozhenko V.A., Pipa V.I. Effect of the optical characteristics of semiconductor resonator structures on the amplitudes of their thermal radiation lines. *Opt. Commun.* 2006. **260**, No 2. P. 607–613. https://doi.org/10.1016/j.optcom.2005.11.008.
12. Baranskii P.I., Klochkov V.P., and Potykevich I.V. *Semiconductor Electronics.* Naukova Dumka, Kiev, 1975 (in Russian).

Authors and CV



Viktor I. Pipa, Ph.D. degree in solid state physics in 1970 from the Institute of Semiconductor Physics. Since 1979, he is Senior Researcher at the Department of Theoretical Physics, V. Lashkaryov Institute of Semiconductor Physics, NAS of Ukraine. He is the author of more than 100 publications. His main research activity is in the coherent thermal radiation and negative luminescence of semiconductor structures.

Anatoliy I. Liptuga, Ph.D. in Physics and Mathematics, Senior Researcher, Head of the Laboratory of UV and IR semiconductor sensors at the V. Lashkaryov Institute of Semiconductor Physics, NAS of Ukraine. The area of scientific interests of Dr. A.I. Liptuga includes physics of plasma phenomena in semiconductors, coherent thermal radiation.