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Optics

Magneto-optic Kerr effect in $Gd_{20}Co_{80}$ alloy

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Abstract. The magneto-optical Kerr effect in $Gd_{20}Co_{80}$ alloy and cobalt thin films has been studied in a broad spectral range applying spectral ellipsometry experimental technique. The results of the experiments showed the complex nature of the complex Kerr angle dispersion curves. A quantum mechanical formalism for degenerate and non-degenerate Landau levels for quasi-free electrons in ferromagnetic material has been developed in order to analyze the experimental data. The equivalence of relations for off-diagonal dielectric tensor elements for non-degenerate Landau levels to the classical case of the motion of quasi-free electrons along circular trajectories in a magnetic field has been theoretically shown. The degenerate Landau levels in this approach are the result of motion of electrons in small confined volumes near rare-earth alloy atoms. Rotation of light polarization occurs in this case due to transitions between subbands having different magnetic quantum numbers. This theoretical approach allowed us to interpret in detail shapes and sign of the complex Kerr angle dispersion curves, which actually include the contributions of optical transitions between degenerate and non-degenerate energy levels. The complex Kerr angle sign is determined by the magnetization magnetic field direction for non-degenerate Landau levels and the Hund rule for degenerate Landau levels.

Keywords: gadolinium cobalt alloy, Landau levels, magneto-optic Kerr effect.

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1. Introduction

The magneto-optical Kerr effect (MOK) is manifested in changing the polarization of light reflected from the magnetized surface. Studying the spectral dependences of the off-diagonal dielectric tensor components that are responsible for this effect proved to be an effective method to determine the magnetic properties of materials. Classical electrodynamics allows one to describe in detail the macroscopic components of the dielectric tensor in a magnetized medium, if taking into account the symmetry of magnetized medium [1]. In fact, the basis of such a theoretical consideration is the analysis of components obtained by decomposition of the dielectric tensor by electrical and magnetic components. For example, the Kerr magneto-optical effect enables to register two components of magnetization in a sample with directions of magnetization parallel to and perpendicular to the plane of light incidence [2]. MOK was registered in the antiferromagnetic material, which allowed the authors [3] to register the magnetic octupole moments.

Quantum-mechanical theory of MOK is based on taking into account the spin-orbit interaction [4] of a single metal atom. In this case, quasi-free electrons in the conduction band are taken into account by introducing a pseudopotential that is actually the sum of Fourier components of the decomposition of orthogonal plane de Broglie electrons in the conduction band. The contribution to the optical conductivity of quasi-free electrons is taken into account by using the Drude theory, which is actually taken as empirically given in this context. The macroscopic characteristics of MOK are related to the direction of the magnetization of the medium, which leads to the appearance of transverse or longitudinal MOK. It is taken into account mainly by calculating the densities of electronic states. The Kubo formula [3] is used to determine the components of the optical conductivity tensor, which is used to obtain macroscopic characteristics of the medium.

The real and complex components of the Kerr angle for gadolinium have positive values within the range of energies 1…5 eV with a strong maximum for the Kerr angle components in the vicinity of energies 4…4.5 eV [5]. For cobalt, negative values were registered for the real and complex component of the Kerr angle in the vicinity of the wavelengths of 350…600 nm [6]. At the same time, there was a change in the sign of the complex component of the Kerr angle in the region close to 2.5 eV for nanometer cobalt film [7] at positive values within the range 1.5…2.5 eV. The sign of the Kerr angle for cobalt changed within the entire range of light wavelengths corresponding to 1…5 eV [8] with changing the direction of perpendicular to the surface of the sample magnetization. Nanostructured by producing hexagonal arrays of subwavelength holes, cobalt films show variation of the MOK effect parameters depending on the hole diameters with the appearance of positive Kerr angle values within the region 3.5…5 eV [9].

The goal of this work is to study experimentally magneto-optical Kerr effect dispersion curves in $Gd_{20}Co_{80}$ alloy as well as to develop the quantum-mechanical formalism to describe the dielectric tensor off-diagonal elements.

2. Theory: off-diagonal dielectric tensor components for degenerate and non-degenerate Landau levels in ferromagnetics

If we have a system with simultaneous acting optical field and DC magnetic field (*e.g.*, spectral ellipsometry of a ferromagnetic sample with crystal potential $U(\vec{r})$) [10], we can present Hamiltonian for this system as:

$$H_0 = \frac{1}{2m_e}\left(p - \frac{e}{c}A(\vec{r},t)\right)^2 + U(\vec{r}). \quad (1)$$

For metal crystal lattice (ignoring $U(\vec{r})$ in the case of quasi-free electrons), we consider optical transition perturbing Eq. (1) Hamiltonian by a polarized along the α axis dipole $d(t)^{\alpha}_{nm}$ induced by a transition between *n* and *m* energy levels. In our case, we have to take into consideration both degenerate and non-degenerate Landau levels formalisms. The models have to take into account both quasi-free electrons motion and electrons interaction with constant magnetic field [11]. Unperturbed by optical field Eq. (1) Hamiltonian can be written using numbers operators (see Eqs (A8) from Appendix) as

$$H_0 = \hbar\omega_B\left(aa^{\dagger} + \frac{1}{2}\right). \quad (2)$$

Eigennumbers of the Hamiltonian (Landau energy levels) can be found introducing positive integers *n*-eigen-numbers of creation and annihilation operators:

$$E_n = (n + 1/2)\hbar\omega_B. \quad (3)$$

Here, *n* is the principal quantum number, $\omega_B = \frac{eB}{m_e c}$ – cyclotron frequency. The dielectric susceptibility can be found as perturbed by induced light dipole transition moments between energy levels Hamiltonian [12]:

$$\chi_{ij}(\omega) = \frac{N}{\hbar}\sum_{m,n}\frac{\Delta_{nm} d^i_{nm} d^j_{mn}}{\omega_{mn} - \omega + i\gamma_{mn}}, \quad (4)$$

where $\Delta_{nm} = \rho_{nn} - \rho_{mm}$ is the difference between relative populations of energy levels, ρ is density matrix.

Applying the known relations for creation and annihilation operators ($a|n\rangle = \sqrt{n}|n-1\rangle$, $a^{\dagger}|n\rangle = \sqrt{n+1}|n+1\rangle$) that were derived in Appendix, we can find off-diagonal tensor elements as a function of these operators. Let us consider one-photon optical transitions ($\Delta n = \pm 1$). Using Eq. (A4) from Appendix, we can derive a new relation for $d^x_{nm} d^y_{mn}$ members from Eq. (4):

$$\left\langle m\left|\left(p_x - \frac{eA_x}{c}\right)\right|n\right\rangle\left\langle n\left|\left(p_y - \frac{eA_y}{c}\right)\right|m\right\rangle =$$
$$= (-i)\sqrt{\frac{m_e\hbar\omega_B}{2}}\left(\sqrt{n}\,\delta_{n,m-1} + \sqrt{n+1}\delta_{n,m+1}\right)\times$$
$$\times\sqrt{\frac{m_e\hbar\omega_B}{2}}\left(\sqrt{n}\,\delta_{n,m-1} - \sqrt{n+1}\delta_{n,m+1}\right) = \quad (5)$$
$$= \frac{im_e\hbar\omega_B}{2}\left((n+1)\delta_{n,m+1} - n\delta_{n,m-1}\right).$$

We can receive expressions for both density matrix $\rho_{n,n}$ and for off-diagonal dielectric tensor elements in a model of all possible optical transitions between Landau energy levels:

$$\sum_{n=0}^{\infty}(n+1)\left(\rho_{n,n} - \rho_{n+1,n+1}\right) =$$
$$\sum_{n=0}^{\infty} n\,\rho_{n,n} + \sum_{n=0}^{\infty}\rho_{n,n} - \sum_{n=0}^{\infty}(n+1)\rho_{n+1,n+1} \equiv \sum_{n=0}^{\infty}\rho_{n,n} = 1,$$

$$\sum_{n=0}^{\infty} n\left(\rho_{n,n} - \rho_{n-1,\,n-1}\right) = \sum_{n=0}^{\infty} n\,\rho_{n,n} - \sum_{n=0}^{\infty}(n+1-1)\rho_{n-1,\,n-1} =$$
$$= \sum_{n=0}^{\infty} n\,\rho_{n,n} - \sum_{n=0}^{\infty}\rho_{n-1,n-1} - \sum_{n=0}^{\infty}(n-1)\rho_{n-1,\,n-1} =$$
$$= -\sum_{n=0}^{\infty}\rho_{n-1,\,n-1} = -1, \quad (6)$$

$$\varepsilon_{yx}(\omega) = -\varepsilon_{xy}(\omega) = 4\pi\chi_{yx}(\omega) = \frac{4\pi N}{\hbar}\sum_{m,n}\frac{\Delta_{nm} d^x_{nm} d^y_{mn}}{\omega_{mn} - \omega + i\gamma_{mn}} =$$
$$= -\frac{4\pi N}{\hbar}\left(\frac{e}{m_e\omega_{mn}}\right)^2\times$$
$$\times\sum_{m,n}\frac{\Delta_{nm}\left(p_x - \frac{eA_x}{c}\right)_{mn}\left(p_y - \frac{eA_y}{c}\right)_{mn}}{\omega_{mn} - \omega + i\gamma_{mn}} =$$

$$= -\frac{4\pi N}{\hbar}\left(\frac{e}{m_e\omega_B}\right)^2\times$$
$$\times\sum_{n}\left(\frac{(n+1)\left(\rho_{n,n} - \rho_{n+1,n+1}\right)}{\omega_B - \omega - i\gamma} - \frac{n\left(\rho_{n,n} - \rho_{n-1,n-1}\right)}{\omega_B + i\gamma + \omega}\right) =$$

$$= \frac{4\pi N}{\hbar}\left(\frac{e}{m_e\omega_B}\right)^2\left(\frac{2\omega_B}{(\omega+i\gamma)^2-\omega_B^2}\right) =$$
$$= \frac{8\pi N}{\hbar\omega_B}\left(\frac{e}{m_e}\right)^2\left(\frac{1}{(\omega+i\gamma)^2-\omega_B^2}\right).$$

Here, $\omega_{mn} = \omega_B$ for $m = n \pm 1$ optical transitions. Thus, we obtain relations for off-diagonal elements of the dielectric tensor, which are similar to the relations within the frames of classical physics with monotonically decreasing dielectric tensor elements for lower light wavelengths [13]. The expression is written under assumption of isotropic alloy having ferromagnetic magnetization along z axis, which results in two nonzero off-diagonal dielectric tensor components. Thus, we obtained an important result for the motion of quasi-free electrons in metal that does not contradict to both classical and quantum-mechanical approaches. Quantum-mechanical description was performed under the condition of Landau levels non-degeneracy.

Degeneration of Landau levels is possible in small volumes of ferromagnets. The energy from Eq. (2) does not depend on the momentum $p_z = \hbar\, k_z$ that is parallel to magnetic field, therefore longitudinal motion is conserved in optical transitions. As a consequence, each Landau level is degenerate because of k_z quantum number. For finite system having S surface maximum numbers of states $n_{\max}$ of energy eigenstates is:

$$n_{\max} \approx \frac{S}{(l_H)^2},$$
$$l_H = \sqrt{\frac{hc}{eB}}. \qquad (7)$$

l_H is the Landau level characteristic length that describes the size of an area with a high probability to find a particle there [14]. For stronger magnetic field, there is a greater degeneracy – electrons are forced to fill a smaller cross-section area as opposite to weak magnetic field, when electron wave functions fill large space areas. For magnetic field 10 Tesla – l_H is equal to 10 nm. This size is of the same order of magnitude as the 20-nm thickness GdCo film from Fig. 1, which indicates the influence of film surfaces on its dispersion curves.

For these confined systems, the Schroedinger equation has to be written with account of geometry inherent to the system, and, as a result, the equation solutions depend on boundary conditions at the surfaces of granules/clusters/nanoparticles. Under assumption of model of quasi-free electrons in central-symmetric electric fields near rare earth atoms, the Schroedinger equation for this geometry can be written in spherical coordinates, similar to those for hydrogen atom, with solutions in the form of products including radial and angular parts:

$$\psi(r,\theta,\varphi) = R(r)Y(\theta,\varphi),$$
$$\hat{L}_z Y(\theta,\varphi) = mY(\theta,\varphi),$$
$$\hat{L}^2 Y(\theta,\varphi) = l(l+1)Y(\theta,\varphi),$$
$$\left(-\frac{\hbar^2}{2m_e}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{L^2}{2m_e r^2}-\frac{eBL_z}{2m_e c}\right)R(r) = ER(r). \qquad (8)$$

Eigenfunctions of L^2 and L_z are usually expressed as spherical harmonic functions $Y_{lm}(\theta,\varphi)$ with eigenvalues $l(l+1)\hbar$ and $m\hbar$, respectively. In this paper, we focus solely on optical Kerr effect rotation as a result of transitions between energy subbands in ferromagnetic intrinsic magnetic field $\Delta E_m = -m\mu_B B$. Magneto-optical Kerr effect can be described as a result of $\Delta m = \pm 1$ transitions. Light circular polarization in the effect is based on the properties of spherical harmonic functions $Y_{lm}(\theta,\varphi)$ [15]. The sign of light rotation for degenerate Landau levels in ferromagnetic alloys is fixed and can be either positive or negative (with a possibility to apply Hund's rule for alloy components). In fact, we consider the MOK effect as a result of the interaction of the angular momentum with macroscopic magnetization similarly to [4], where this interaction is considered at the level of a single atom (spin-orbit interaction).

The most significant features of degenerate and non-degenerate Landau levels models in ferromagnets are given in Table.

From a purely experimental view point, Eq. (6) differs from Eq. (8) for electrons in a spherically-symmetric field by the absence of resonant frequencies with symmetric/antisymmetric shapes of imaginary/real components of the Kerr complex angle on dispersion dependences. The case of Eqs (6) results in monotonically decreasing the $\theta(\omega)$ and $\eta(\omega)$ curves.

3. Experimental

The magneto-optical study of $Gd_{20}Co_{80}$ alloy films was carried out by studying the linear magneto-optical Kerr effect in a polar geometry. This effect implies rotation of the polarization plane and the appearance of ellipticity of the light reflected from a medium magnetized perpendicularly to the surface. We measured the complex Kerr angle in a wide range of the spectrum λ = 0.24…1.0 μm by using the Woollam M-2000 spectral ellipsometer. $Gd_{20}Co_{80}$ alloy films with thicknesses from 20 to 100 nm were deposited using the ion-plasma method at a constant current in vacuum at the pressure of 10^{-3} Pa on glass water-cooled substrates.

With the spherical symmetry of the spatial restriction of the motion of quasi-free electrons, we can consider optical transitions by applying the stationary quantum-mechanical perturbation theory, which leads to the relation (4). In this case, Kerr angles have an additional physical meaning – they are functions of the matrix elements of optical transitions with the implementation of the selection rules $\Delta m = \pm 1$. If we consider spherically-symmetric solutions of the Schrödinger equation (Eq. (8)), it contains a component in the form of spherical functions $Y_{lm}(\theta,\varphi)$. As a consequence, the

Table. Magneto-optical Kerr effect in rare earth and *d*-metal alloys.

Landau levels in ferromagnets	Degenerate states	Non-degenerate states
Symmetry	Confined spherical geometry in the vicinity of atoms of rare earth and *d*-metals	Quasi-free electrons in infinite metal system with weak magnetization
Schroedinger equation solutions	Spherical harmonic functions	Landau levels wave functions
Quantum numbers selection rules responsible for polarization plane rotation	Transitions between energy subbands with $\Delta m = \pm 1$	Transitions between Landau levels with $\Delta n = \pm 1$
Sign of light rotation	Depends on Δm sign	Depends on magnetic field direction
Optical dispersion properties	Changing the sign of the derivative of the real part of the Kerr angle in the vicinity of narrow extremes.	Monotonic dependences of the real and imaginary components of the Kerr angle similarly to classical model of quasi-free electrons motion in magnetic field.

projection of the dipole moments of the optical transitions on x and y axes are connected and shifted in time by $\mp\pi/2$ at $\Delta m = \pm 1$. The dielectric tensor elements $\varepsilon_{xy}(\omega)$ and $\varepsilon_{xx}(\omega)$ determined applying Eq. (4) are related similarly:

$$d^y_{nm} d^x_{mn} = e\langle n|y|m\rangle d^x_{mn} = -ie\Delta m\langle n|x|m\rangle d^x_{mn} =$$
$$= -i\Delta m d^x_{nm} d^x_{mn},$$
$$i\varepsilon_{xy}(\omega) \equiv \varepsilon_{xx}(\omega). \quad (9)$$

Thus, if we consider the dispersion dependence of the complex refractive index $n + ik$ in the narrow spectral range of such an optical transition with the frequency $\omega = \omega_0$, we can write as a consequence of Eqs (4) and (9):

$$\theta + i\eta = \frac{i\varepsilon_{xy}}{(\varepsilon_{xx} - 1)\sqrt{\varepsilon_{xx}}},$$
$$\theta(\omega) + i\eta(\omega) \equiv i\varepsilon_{xy}(\omega) \equiv \varepsilon_{xx}(\omega) \equiv (\omega - \omega_0 + i\gamma)^{-1}. \quad (10)$$

The first relationship from Eq. (10) is taken from [13]. In fact, we are talking about optical transitions between states in the vicinity of alloy atoms at $\Delta m = \pm 1$ with antisymmetric and symmetric dependences for correspondingly real $\theta(\omega)$ and complex $\eta(\omega)$ parts of the Kerr angle near the resonant frequency ω_0. At the qualitative level, Eq. (10) should be pronounced in the dispersion dependences as local extrema in $\eta(\lambda)$ curves with a simultaneous change in the sign of the derivative in the dependences $\theta(\lambda)$. Eq. (10) can be used for analyzing the qualitative experiments to identify these transitions. Optical transitions between non-degenerate Landau levels that result from electrons in magnetic field and move at some distance from the alloy atoms also contribute to the total magneto-optical effect.

Fig. 1 shows the dispersion dependences of complex Kerr angles for $Gd_{20}Co_{80}$ alloy films of different thickness values. As can be seen from Fig. 1a, $\eta(\lambda)$ curves have a strong absorption maximum at the energy 1.45 eV and a weak maximum in the range of 3.2 eV

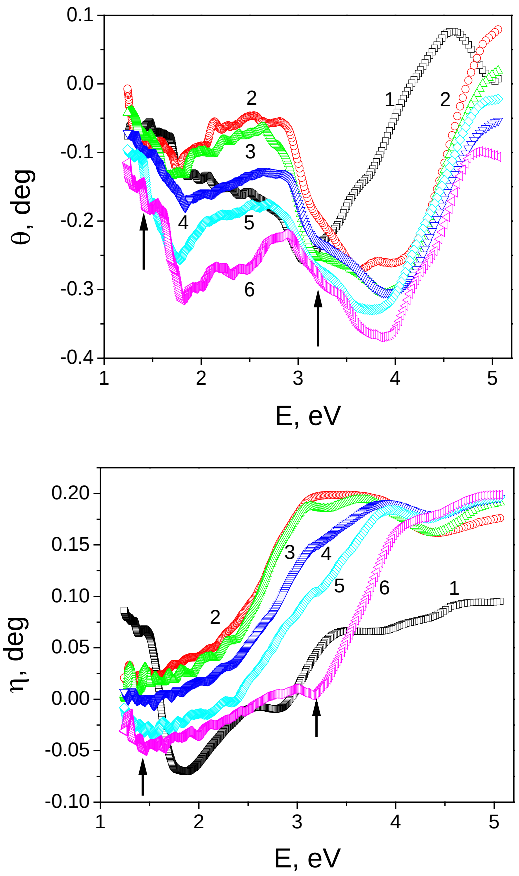

Fig. 1. Complex Kerr angles in $Gd_{20}Co_{80}$ thin films of different thickness (*1* – 20 nm, *2* – 40, *3* – 50, *4* – 60, *5* – 80, *6* – 100).

(indicated with arrows), which correspond to the simultaneous change in the sign of $\theta(\lambda)$ derivatives at these two energy values. The maximum located at 3.2 eV (Fig. 1a) is most likely associated with gadolinium atoms in the sample alloy. These transitions for gadolinium atoms must have weaker cross-sections, because its valence electrons originate from inner electronic shells. It is also important to note the change in $\eta(\lambda)$ signs in Fig. 1 at high values of light energies. In this case,

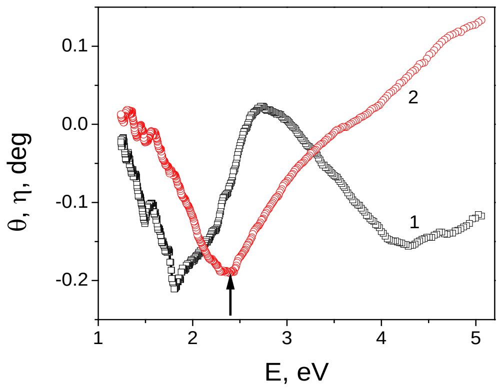

Fig. 2. Complex magneto-optical Kerr effect of electron-beam deposited Co film with the thickness 35 nm (*1* – θ(*E*), *2* – η(*E*)).

we consider optical transitions that depend on the magnetization direction in a ferromagnet. The 1.45 and 3.2 eV absorption lines in Fig. 1 of the $Gd_{20}Co_{80}$ alloy are not typical for the spectral curves of gadolinium and cobalt alone. Thus, gadolinium is characterized by the absorption line at 2.6 eV [5]. In the case of cobalt (Fig. 2), we are talking about a band in the vicinity of 2.4 eV similarly to the results in [9]. In the case of the $Gd_{20}Co_{80}$ alloy, its band gap obviously does not coincide with the energy gaps of neither gadolinium nor cobalt.

If we consider the experimental dispersion dependences for the Kerr angle in the cobalt film in Fig. 2, we can see that the curve of the imaginary part of the complex Kerr angle changes its sign to positive within the spectral range above 3.5 eV. MOK in cobalt is well studied with negative values of the Kerr angle component [6, 16]. Thus, Fig. 2 dependences can be explained similarly to $Gd_{20}Co_{80}$ alloy case as results of presence of optical transitions with arbitrary direction of magnetization vector which influences θ(λ) and η(λ) signs.

Dispersive curves in Fig. 1 also depend on the thickness of thin films similarly to the results in [17]. In MOK effect, the thin film thickness can drastically change electrons interaction with magnetization field. It can happen when the film thickness approaches to the Landau levels characteristic length l_H from Eq. (7). As the film thickness decreases to this point, the spherically symmetrical boundary conditions for the Schroedinger equation (Eq. (8)) become not valid anymore.

4. Conclusions

We have performed a theoretical analysis of motion inherent to quasi-free electrons in a magnetized ferromagnetic alloy by applying a quantum-mechanical formalism to describe optical transitions between Landau levels. The experimental part of the research including spectral-ellipsometric studies of the complex Kerr angle in thin films of $Gd_{20}Co_{80}$ alloy of different thickness values as well as a thin film of cobalt. Being based on this study, we can draw the following conclusions:

1. Motion of quasi-free electrons in a magnetic field inside infinite metal volumes with weak magnetization results in nonzero dielectric tensor off-diagonal elements, *i.e.*, to nonzero complex Kerr angles. For non-degenerate Landau energy levels, the relations between the values of the off-diagonal elements of the tensor are in fact similar to the classical formalism, which describes circular motion of electrons in magnetic field. The sign of the Kerr angle (left- or right-circular reflected light) in a non-degenerate system of Landau levels depends on the direction of the magnetic field perpendicular to the sample surface, *i.e.*, can vary its sign depending on the magnetization of the sample.

2. In the case of degeneracy of Landau levels due to the motion of electrons in small spatial volumes in the vicinity of *d*-metal atoms or rare-earth atoms of ferromagnetic alloys, the solutions of the Schroedinger equation are written as spherical functions due to the obvious spherical symmetry of the system. The nonzero Kerr effect in this formalism occurs due to the greater number of transitions between the electronic levels of the corresponding atom with a change in the magnetic quantum number +1 or –1. In this case, the sign of the Kerr angle is constant and is defined by the electronic structure of the corresponding atoms of the metal alloy (in our case, cobalt and gadolinium).

3. The dispersion dependences of the optical constants of the ferromagnetic alloy contain two components due to optical transitions between both non-degenerate and degenerate Landau levels. For the latter case, we have simultaneously obtained rather narrow extremes of the imaginary part of the complex Kerr angle and changing the real part of the Kerr angle sign. These extremes arise due to motion of electrons in confined volumes near the alloy atoms. Quasi-free electrons in the infinite system model contribute to the dispersion curves as a monotonic offset, which can be either positive or negative depending on directions of the magnetic field in a sample. In this case, the magneto-optical Kerr effect can be described within the framework of classical model for quasi-free electrons circular motion in the magnetic field.

Appendix A.

Let us find relationships between $p-\frac{eA}{c}$ member from Eq. (1) and creation $a^{\dagger}$ and annihilation a operators for quasi-free electrons of ferromagnetic with intrinsic magnetic field directed along Z axis. We would like here to write the equations for Landau levels similar to the known form for quantum harmonic oscillator to simplify equations for off-diagonal dielectric tensor components.

1. Let us find first the relationship between vector potential A, magnetic field $B = B_z$ and angular momentum projection L_z for unperturbed H_0 Hamiltonian (taken from Eq. (1)) for quasi-free electrons:

$$H_0 = \frac{p^2}{2m_e} - \frac{eAp}{m_e c} + \frac{(eA)^2}{2m_e c^2}\,. \qquad \text{(A1)}$$

Let us choose the vector potential as $A=(-By/2,\ Bx/2,\ 0)$ and find a relation between Ap and L_z:

$$\vec{\nabla}\times\vec{A}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ -By/2 & Bx/2 & 0\end{vmatrix}=$$

$$=A_z\vec{k}-(B/2-(-B/2))\vec{k}=B\vec{k},$$

$$\vec{L}=\vec{r}\times\vec{p},$$

$$L_z=xp_y-yp_x,$$

$$Ap=(-By/2,\ Bx/2,\ 0)(p_x,p_y,p_z)= \\ =B/2\,(p_y x-p_x y)=L_z B/2. \qquad (A2)$$

Eq. (A2) describes interaction of electron with the magnetic field in terms of angular momentum projection similar to the energy term in Eq. (8) $\left(-\frac{eBL_z}{2m_e c}=-m\mu_B B\right)$ there for spherical harmonic functions solutions.

2. Let us find a connection between the dipole moment d, unperturbed Hamiltonian H_0 and $p-\frac{eA}{c}$ by writing the commutator of coordinate with Eq. (A1) H_0 Hamiltonian. We consider only kinetic energy and interaction with the magnetic field Eq. (A1) parts of H_0 because the last term of expansion $\frac{(eA)^2}{2m_e c^2}$ is a constant function of coordinates:

$$[Ap,x]\psi=[xBp_y,x]\psi-[yBp_x,x]\psi=$$

$$=-i\hbar B\left(x\frac{\partial}{\partial y}(x\psi)-x^2\frac{\partial}{\partial y}\psi\right)-[yBp_x,x]\psi=$$

$$=-[yBp_x,x]\psi=-B[L_z,x]\psi/2,$$

$$[p^2,x]\psi=[p_x^2,x]\psi=(-i\hbar)^2\left(\frac{\partial^2}{\partial x^2}(x\psi)-x\frac{\partial^2}{\partial x^2}\psi\right)=$$

$$=-\hbar^2\left(\frac{\partial\psi}{\partial x}+x\frac{\partial^2}{\partial x^2}\psi+\frac{\partial\psi}{\partial x}-x\frac{\partial^2}{\partial x^2}\psi\right)=-2i\hbar p_x\psi,$$

$$[H_0,x]=\left[\frac{p^2}{2m_e},x\right]-\left[\frac{eAp}{m_e c},x\right]=\left[\frac{p_x^2}{2m_e},x\right]-\frac{eB}{2m_e c}[L_z,x]=$$

$$=\frac{i\hbar p_x}{m_e}-\frac{eB}{2m_e c}(i\hbar y)=-\frac{i\hbar}{m_e}\left(p_x+\frac{eB}{2c}y\right)=$$

$$=-\frac{i\hbar}{m_e}\left(p_x-\frac{eA_x}{c}\right). \qquad (A3)$$

Similarly we can prove $[H_0,y]=\frac{-i\hbar}{m_e}\left(p_y-\frac{eA_y}{c}\right)$.

As a result, the dipole moments d induced by light between two states m and n can be expressed in terms of $p-\frac{eA}{c}$ member by writing first the commutator for x axis (it can be written similarly to y axis, too).

$$\left\langle m\left|\left(p_x-\frac{eA_x}{c}\right)\right|n\right\rangle=\left\langle m\left|\frac{im_e}{\hbar}[H_0,x]\right|n\right\rangle=$$

$$=\frac{im_e}{\hbar}\left((\psi_m H_0)x\psi_n^*-\psi_m x(H_0\psi_n^*)\right)=$$

$$=\frac{im_e}{\hbar}(E_m-E_n)\langle m|x|n\rangle,$$

$$\left\langle m\left|\left(p-\frac{eA}{c}\right)\right|n\right\rangle=\frac{im_e\omega_{mn}}{e}\langle m|d|n\rangle. \qquad (A4)$$

3. Let us find the commutators of $p_x-\frac{eA_x}{c}$, $p_y-\frac{eA_y}{c}$ operators with the vector-potential A that depends only on X and Y coordinates.

$$[p_x,A_y]\psi=\left(-i\hbar\frac{\partial}{\partial x}\right)(A_y\psi)-(A_y)\left(-i\hbar\frac{\partial\psi}{\partial x}\right)=$$

$$=\left(-i\hbar\frac{\partial A_y}{\partial x}\right)(\psi)+(A_y)\left(-i\hbar\frac{\partial\psi}{\partial x}\right)-(A_y)\left(-i\hbar\frac{\partial\psi}{\partial x}\right)=$$

$$=-i\hbar\frac{\partial A_y}{\partial x}\psi,$$

$$[A_x,p_y]\psi=i\hbar\left(\frac{\partial A_x}{\partial y}\right)\psi,$$

$$\left[p_x-\frac{e}{c}A_x,\,p_y-\frac{e}{c}A_y\right]=-\frac{e}{c}\left([p_x,A_y]+[A_x,p_y]\right)=$$

$$=-\frac{e}{c}(-i\hbar)\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)=\frac{ie\hbar}{c}\left(\vec{\nabla}\times\vec{A}\right)_z=i\hbar\frac{eB}{c}=const. \qquad (A5)$$

4. Let us introduce a and $a^\dagger$ operators ($\omega_B=\frac{eB}{m_e c}$ cyclotron frequency):

$$a=\frac{\left(p_x-\frac{eA_x}{c}\right)+i\left(p_y-\frac{eA_y}{c}\right)}{\sqrt{2m_e\hbar\omega_B}},$$

$$a^\dagger=\frac{\left(p_x-\frac{eA_x}{c}\right)-i\left(p_y-\frac{eA_y}{c}\right)}{\sqrt{2m_e\hbar\omega_B}}. \qquad (A6)$$

Let us prove the equation (A6) operators are creation and annihilation operators for unperturbed Hamiltonian similarly to the quantum harmonic oscillator case. In order to prove it, let us find commutators of $p_x-\frac{eA_x}{c}$, $p_y-\frac{eA_y}{c}$ operators with creation $a^\dagger$ and annihilation a operators:

$$\left[a,a^{\dagger}\right]=\frac{c}{2\hbar eB}\left[\left(p_x-\frac{eA_x}{c}\right)+i\left(p_y-\frac{eA_y}{c}\right),\atop\left(p_x-\frac{eA_x}{c}\right)-i\left(p_y-\frac{eA_y}{c}\right)\right]=$$

$$=\frac{c}{2\hbar eB}\left[-i\left[\left(p_x-\frac{eA_x}{c}\right),\left(p_y-\frac{eA_y}{c}\right)\right]+\atop+i\left[\left(p_y-\frac{eA_y}{c}\right),\left(p_x-\frac{eA_x}{c}\right)\right]\right]= \qquad (A7)$$

$$=\frac{c}{2\hbar eB}(-2i)\left(i\hbar\frac{e}{c}B_z\right)=1,$$

$$p_x-\frac{eA_x}{c}=\sqrt{\frac{m_e\hbar\omega_B}{2}}\left(a+a^{\dagger}\right),$$

$$p_y-\frac{eA_y}{c}=-i\sqrt{\frac{m_e\hbar\omega_B}{2}}\left(a-a^{\dagger}\right).$$

5. As a result, Eq. (A1) Hamiltonian of the system can be written similarly to quantum harmonic oscillator equations:

$$H_0=\frac{1}{2m_e}\left(\left(p_x-\frac{eA_x}{c}\right)^2+\left(p_y-\frac{eA_y}{c}\right)^2\right)=$$

$$=\frac{1}{2m_e}\frac{1}{2}\begin{pmatrix}\left(\left(p_x-\frac{eA_x}{c}\right)+i\left(p_y-\frac{eA_y}{c}\right)\right)\times\\ \times\left(\left(p_x-\frac{eA_x}{c}\right)-i\left(p_y-\frac{eA_y}{c}\right)\right)+\\ +\left(\left(p_x-\frac{eA_x}{c}\right)-i\left(p_y-\frac{eA_y}{c}\right)\right)\times\\ \times\left(\left(p_x-\frac{eA_x}{c}\right)+i\left(p_y-\frac{eA_y}{c}\right)\right)\end{pmatrix}=$$

$$=\frac{1}{2}\hbar\omega_B\left(aa^{\dagger}+a^{\dagger}a\right)=\hbar\omega_B\left(a^{\dagger}a+1/2\right),$$

$$a\left|n\right\rangle=\sqrt{n}\left|n-1\right\rangle,$$

$$a^{\dagger}\left|n\right\rangle=\sqrt{n+1}\left|n+1\right\rangle. \qquad (A8)$$

Eqs (A4) and (A8) allow us to write equations for optical transitions between Landau levels by using $a^{\dagger}$ and a operators.

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Магнітооптичний ефект Керра у сплаві $Gd_{20}Co_{80}$

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Анотація. Магнітооптичний ефект Керра у тонких плівках сплаву $Gd_{20}Co_{80}$ та кобальту вивчено за допомогою експериментальної методики спектральної еліпсометрії у широкому спектральному інтервалі. Результати експериментів показали складний характер дисперсійних залежностей комплексного кута Керра. Для аналізу експериментальних даних було розвинуто квантовомеханічний формалізм вироджених і невироджених рівнів Ландау для квазівільних електронів у феромагнітному матеріалі. Для невироджених рівнів Ландау було теоретично показано еквівалентність співвідношень для недіагональних тензорних елементів діелектричної проникності класичному випадку руху квазівільних електронів вздовж циркулярних траєкторій у магнітному полі. Вироджені рівні Ландау при даному підході є результатом руху електронів у малих обмежених об'ємах біля рідкоземельних атомів сплаву. При цьому поворот поляризації світла відбувається внаслідок переходів між енергетичними рівнями з різними значеннями магнітного квантового числа. Даний теоретичний підхід дозволив у деталях проінтерпретувати хід та знак дисперсійних кривих комплексного кута Керра, які фактично включають внески від оптичних переходів між виродженими та невиродженими енергетичними рівнями. Знак комплексного кута Керра визначається напрямком магнітного поля намагніченості для невироджених рівнів Ландау та правилом Хунда для вироджених рівнів Ландау.

Ключові слова: сплав гадоліній кобальт, рівні Ландау, магнітооптичний ефект Керра.