Optics

Suppressing internet bottleneck with Kudryashov's extended and exotic form of self-phase modulation structure having fractional temporal evolution

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Abstract. This paper optimistically and aggressively targeted to control the internet bottleneck effect with Kudryashov's extended and exotic form of self-phase modulation structure by using the modified simple equation method. This approach disappointingly failed to retrieve the solutions for bright soliton that serves as the information carrier bits. The model was considered with fractional temporal evolution. The failure of the integration scheme to retrieve bright soliton solutions teaches a stern lesson in this paper about the adopted non-robust integration algorithm.

Keywords: solitons, Kudryashov's method, fractional temporal evolution.

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1. Introduction

The Internet bottleneck effect is a growing problem that has engulfed the telecommunications industry [1–4]. The frustrated customers of internet service providers are expressing their need and demand for a stable internet traffic flow that is uninterrupted. This is an absolute necessity in the modern technologically dominated world. Thus, there are several measures [5–8] that have been adopted to address this issue. One of the measures and means that contains the problem is the consideration of fractional temporal evolution [9–12] as opposed to linear temporal evolution [13]. This can slow the evolution of pulses and the Internet traffic can have a regular speed in the opposite direction. This can create a traffic flow effect at internet traffic control junction points, namely in HI, USA.

Several factors dictate the flow of soliton pulses in an optical fiber. Apart from the temporal evolution of pulses, one of the essential factors is the self–phase modulation (SPM) structure [14]. While the most visible form of SPM is described by the Kerr law, this paper considers the extended and exotic form of SPM that was first proposed by Kudryashov less than a decade ago [15]. The modified simplest equation approach applies the model and retrieves the soliton solution [1–6]. It will be observed that the approach fails to recover the bright soliton solutions that are the essential structure of the pulses or the information carrier bits. So, in other words, it will be shown that the integration approach adopted in the works is an epic failure to successfully recover the bright soliton solution to the model with this SPM structure. The details are very clearly presented and the results are exhibited in the rest of the paper.

1.1. Governing model

This study investigates the nonlinear Schrödinger equation with fractional temporal evolution, with Kudryashov's extended exotic self-phase modulation structure [1]:

$$i\frac{\partial^\beta q}{\partial t^\beta} + a q_{xx} + (p_1|q|^n + p_2|q|^{2n} + p_3|q|^{3n} + p_4|q|^{4n})q + \\ +\{p_5(|q|^n)_{xx} + p_6(|q|^{2n})_{xx}\}q = 0, \quad 0 < \beta \le 1, \tag{1}$$

where $q(x, t)$ represents the wave profile and $\frac{\partial^\beta(.)}{\partial t^\beta}$ denotes the conformable derivative, with n being the nonlinearity parameter. The coefficients p_i for $i = 0,1,2,3,4$ describe the nonlinearity effects, contributing to the SPM. Furthermore, the coefficients p_5 and p_6 characterize a generalized non-local formulation of the refractive index.

Definition 1.1. Let $p:(0,\infty) \to R$. The conformable derivative of order β can be introduced as follows:

$$L_\beta(p)(x) = \lim_{a\to 0}\frac{p(x + ax^{1-\beta}) - p(x)}{a}, \tag{2}$$

for all $x > 0$ and $\beta \in (0,1]$ [2].

2. Mathematical preliminaries

Here, we start with inserting the following wave transform into the equation (1):

$$q(x,t) = U(\varsigma)e^{i\vartheta(x,t)}, \\ \varsigma = x - v\frac{t^\beta}{\beta}, \vartheta(x,t) = \rho - kx + \omega\frac{t^\beta}{\beta}, \tag{3}$$

where ρ represents the phase center, k stands for the soliton frequency, and ω denotes the wave frequency. Upon incorporating the transformations from equation (3) into equation (1), the resulting expression is as follows:

$$bU'' + 2bp_6 nU^{2n}U'' + p_5 nU^n U'' + \\ p_5 n(n-1)U^{n-1}U'^2 (2bk^2 + \omega)U + \\ 2p_6 n(2n-1)U^{2n-1}U'^2 - p_1 U^{n+1} - \\ p_2 U^{2n+1} - p_4 U^{4n+1} - p_3 U^{3n+1} = 0, \tag{4}$$

and the imaginary part

$$(v + 2bk)U' = 0. \tag{5}$$

From the above equation, the velocity has the following form:

$$v = -2bk. \tag{6}$$

Setting

$$U(\varsigma) = V^{\frac{1}{n}}(\varsigma). \tag{7}$$

Thus, from equations (6) and (7), we obtain

$$a(1-n)V'^2 + naVV'' + 2c_6 n^2 aV''V^3 \\ +2c_6 n^2 aV'^2 V^2 + c_5 n^2 V''V^2 - \\ n^2(2ak^2 + \omega)V^2 + c_1 n^2 V^3 + \\ c_2 n^2 V^4 + c_3 n^2 V^5 + c_4 n^2 V^6 = 0, \tag{8}$$

3. Modified simplest equation approach

In this section, we present several novel conformable optical soliton solutions for the current model, derived using the modified simplest equation method. We assume that the solution to equation (8) can be expressed as the following series:

$$V(\varsigma) = \sum_{i=0}^{M} a_i G(\varsigma)^i, \tag{9}$$

where a_0, a_1,…, a_M are unknown constants, and M is a balancing parameter. In equation (8), implementing the balancing principle between V^3V'' and V^6 leads to $M = 1$. Here, from the equation (9) the following is obtained:

$$V(\varsigma) = a_0 + a_1 G(\varsigma), \tag{10}$$

where $G(\varsigma)$ satisfies the following equation:

$$G(\varsigma)' = h_0 + G(\varsigma)^2. \tag{11}$$

Now, the solutions of equation (8) with parameter l can be defined as follows [3]:

$$G_1(\varsigma) = -\sqrt{-h_0}\tanh\left[\sqrt{-h_0}(\varsigma + l)\right], \tag{12}$$

$$G_2(\varsigma) = -\sqrt{-h_0}\coth\left[\sqrt{-h_0}(\varsigma + l)\right], \tag{13}$$

$$G_3(\varsigma) = \sqrt{-h_0}\left(-\tanh\left[2\sqrt{-h_0}(\varsigma + l)\right] \\ \pm i sech\left[2\sqrt{-h_0}(\varsigma + l)\right]\right), \tag{14}$$

$$G_4(\varsigma) = \sqrt{-h_0}\left(-\coth\left[2\sqrt{-h_0}(\varsigma + l)\right] \\ \pm csch\left[2\sqrt{-h_0}(\varsigma + l)\right]\right), \tag{15}$$

$$G_5(\varsigma) = -\frac{\sqrt{-h_0}}{2}\left(\tanh\left(\tanh\left[\frac{\sqrt{-h_0}}{2}(\varsigma + l)\right] \pm \coth\left[\frac{\sqrt{-h_0}}{2}(\varsigma + l)\right]\right)\right), \tag{16}$$

with $h_0 < 0$.

Substituting equations (10) and (11) into equation (8) yields a polynomial expressed in terms of powers of $G(\varsigma)$. Then we arrange terms with similar powers and set each corresponding coefficient to zero. This process generates the following system of algebraic equations:

$$(G(\varsigma))^0: -bk^2n^2a_0^2 - n^2wa_0^2 + ba_1^2h_0^2 - \\ -bna_1^2h_0^2 + n^2a_0^3p_1 + n^2a_0^4p_2 + \\ +n^2a_0^5p_3 + n^2a_0^6p_4 + 2n^2a_0^2a_1^2h_0^2p_6 = 0, \tag{17}$$

$$(G(\varsigma))^1: -2bk^2n^2a_0a_1 - 2n^2\omega a_0a_1 + 2bna_0a_1h_0 + \\ +3n^2a_0^2a_1p_1 + 4n^2a_0^3a_1p_2 + 5n^2a_0^4a_1p_3 + \\ +6n^2a_0^5a_1p_4 + 2n^2a_0^2a_1h_0p_5 + \\ +4n^2a_0^3a_1h_0p_6 + 4n^2a_0a_1^3h_0^2p_6 = 0, \tag{18}$$

$$(G(\varsigma))^2: -bk^2n^2a_1^2 - n^2wa_1^2 + 2ba_1^2h_0 + \\ 3n^2a_0a_1^2p_1 + 6n^2a_0^2a_1^2p_2 + 10n^2a_0^3a_1^2p_3 + \\ +15n^2a_0^4a_1^2p_4 + 4n^2a_0a_1^2h_0p_5 + \\ +16n^2a_0^2a_1^2h_0p_6 + 2n^2a_1^4h_0^2p_6 = 0, \tag{19}$$

$$(G(\varsigma))^3: 2bna_0a_1 + n^2a_1^3p_1 + 4n^2a_0a_1^3p_2 + 10n^2a_0^2a_1^3p_3 \\ +20n^2a_0^3a_1^3p_4 + 2n^2a_0^2a_1p_5 + 2n^2a_1^3h_0p_5 \\ +4n^2a_0^3a_1p_6 + 20n^2a_0a_1^3h_0p_6 = 0, \tag{20}$$

$$(G(\varsigma))^4: ba_1^2 + bna_1^2 + n^2a_1^4p_2 + 5n^2a_0a_1^4p_3 + \\ +15n^2a_0^2a_1^4p_4 + 4n^2a_0a_1^2p_5 + \\ 14n^2a_0^2a_1^2p_6 + 8n^2a_1^4h_0p_6 = 0, \tag{21}$$

$$(G(\varsigma))^5: n^2 a_1^5 p_3 + 6n^2 a_0 a_1^5 p_4 + 2n^2 a_1^3 p_5 + 16n^2 a_0 a_1^3 p_6 = 0, \tag{22}$$

$$(G(\varsigma))^6: n^2 a_1^6 p_4 + 6n^2 a_1^4 p_6 = 0. \tag{23}$$

The following results are acquired by solving this system:

Result–1:

$$a_0 = -\frac{2b(2+n)h_0}{n^2(p_1 - 4h_0 p_5)}, \qquad a_1 = \pm 2b(2+n)\sqrt{\frac{-h_0}{n^4(p_1 - 4h_0 p_5)^2}}, \qquad k = \pm\frac{\sqrt{-n^2\omega - 4bh_0}}{\sqrt{bn}},$$

$$p_2 = \frac{\frac{3n^6(1+n)p_1^2}{h_0} + 12n^6(4+n)p_1 p_5 - 48n^6(5+2n)h_0 p_5^2 + \frac{128b^3(2+n)^4 h_0^2 p_4}{(p_1 - 4h_0 p_5)^2}}{12bn^4(2+n)^2}, \tag{24}$$

$$p_3 = \frac{\frac{3n^6 p_1^2 p_5}{h_0} - 24n^6 p_1 p_5^2 + 8h_0\left(6n^6 p_5^3 + \frac{5b^3(2+n)^3 p_4}{p_1 - 4h_0 p_5}\right)}{6b^2 n^2 (2+n)^2}, \qquad p_6 = \frac{2b^2(2+n)^2 h_0 p_4}{3n^4(p_1 - 4h_0 p_5)^2}.$$

By substituting the functions $G_j(\varsigma)$ for $j = 1,\ldots,5$ into equation (10) and using equations (3), (6), and (7), we can find the solutions to equation (1). This involves inserting Result-1 into $q_j(x,t)$, where $j = 1,\ldots,5$, to obtain the following soliton solutions:

$$q_1(x,t) = \left(\frac{-2b(2+n)h_0}{n^2(p_1 - 4h_0 p_5)} \pm \frac{2b(2+n)h_0 \tanh\left[\sqrt{-h_0}(l+\varsigma)\right]}{\sqrt{n^4(p_1 - 4h_0 p_5)^2}}\right)^{\frac{1}{n}} e^{i\left(\frac{t^\beta \omega}{\beta} \pm \frac{x\sqrt{-n^2\omega - 4bh_0}}{\sqrt{bn}} + \rho\right)}, \tag{25}$$

which is a dark soliton.

$$q_2(x,t) = \left(\frac{-2b(2+n)h_0}{n^2(p_1 - 4h_0 p_5)} \mp \frac{2bh_0(2+n)\coth\left[\sqrt{-h_0}(l+\varsigma)\right]}{\sqrt{n^4(p_1 - 4h_0 p_5)^2}}\right)^{\frac{1}{n}} e^{i\left(\frac{t^\beta \omega}{\beta} \pm \frac{x\sqrt{-n^2\omega - 4bh_0}}{\sqrt{bn}} + \rho\right)}, \tag{26}$$

and this is a singular soliton.

$$q_3(x,t) = \left(a_0 \mp \frac{2b(2+n)h_0\left(i\,\mathrm{sech}\left[2\sqrt{-h_0}(l+\varsigma)\right] + \tanh\left[2\sqrt{-h_0}(l+\varsigma)\right]\right)}{\sqrt{n^4(p_1 - 4h_0 p_5)^2}}\right)^{\frac{1}{n}} e^{i\left(\frac{t^\beta \omega}{\beta} \pm \frac{x\sqrt{-n^2\omega - 4bh_0}}{\sqrt{bn}} + \rho\right)}, \tag{27}$$

which is a complexiton solution.

$$q_4(x,t) = \left(a_0 \mp \frac{2b(2+n)h_0\left(csch\left[2\sqrt{-h_0}(l+\varsigma)\right] - \coth\left[2\sqrt{-h_0}(l+\varsigma)\right]\right)}{\sqrt{n^4(p_1 - 4h_0 p_5)^2}}\right)^{\frac{1}{n}} e^{i\left(\frac{t^\beta \omega}{\beta} \pm \frac{x\sqrt{-n^2\omega - 4bh_0}}{\sqrt{bn}} + \rho\right)}, \tag{28}$$

which is a singular–singular straddled soliton.

$$q_5(x,t) = \left(a_0 \mp \frac{2b(2+n)h_0\left(\coth\left[\frac{\sqrt{-h_0}}{2}(l+\varsigma)\right] + \tanh\left[\frac{\sqrt{-h_0}}{2}(l+\varsigma)\right]\right)}{\sqrt{n^4(p_1 - 4h_0 p_5)^2}}\right)^{\frac{1}{n}} e^{i\left(\frac{t^\beta \omega}{\beta} \pm \frac{x\sqrt{-n^2\omega - 4bh_0}}{\sqrt{bn}} + \rho\right)} \tag{29}$$

where $\varsigma = x - \frac{2\sqrt{b}t^\beta\sqrt{-n^2\omega - 4bh_0}}{n\beta}$ is a dark-singular straddled soliton.

Result–2:

$$a_1 = \frac{\sqrt{2a_0(b(2+n) - 2n^2 a_0 p_5)}}{\pm n\sqrt{p_1}}, k = \pm\sqrt{\frac{2a_0(bp_1 + n^2\omega p_5) - b(2+n)\omega}{b(b(2+n) - 2n^2 a_0 p_5)}}, h_0 = \frac{-n^2 a_0 p_1}{2b(2+n) - 4n^2 a_0 p_5},$$

$$p_6 = \frac{a_0 p_4(-b(2+n) + 2n^2 a_0 p_5)}{3n^2 p_1}, p_2 = \frac{-9p_1 + 16a_0^3 p_4 + \frac{3b(5+2n)p_1}{b(2+n) - 2n^2 a_0 p_5}}{6a_0},$$

$$p_3 = \frac{10b(2+n)a_0^2 p_4 + n^2(3p_1 - 20a_0^3 p_4)p_5}{-3b(2+n)a_0 + 6n^2 a_0^2 p_5}. \tag{30}$$

By substituting the functions $G_j(\varsigma)$ for $j = 1,\ldots,5$ into equation (10) and using equations (3), (6), and (7), we can find the solutions to equation (1). This involves inserting Result-2 into $q_j(x,t)$, where $j = 6,\ldots,10$, to obtain the following soliton solutions:

$$q_6(x,t) = \left(a_0 \pm \frac{\sqrt{\frac{2n^2 a_0^2 p_1(b(2+n) - 2n^2 a_0 p_5)}{2b(2+n) - 4n^2 a_0 p_5}}\tanh\left[\sqrt{\frac{n^2 a_0 p_1}{2b(2+n) - 4n^2 a_0 p_5}}(l+\varsigma)\right]}{n\sqrt{p_1}}\right)^{\frac{1}{n}} e^{i\left(\omega\frac{t^\beta}{\beta} \pm \sqrt{\frac{2a_0(bp_1 + n^2\omega p_5) - b(2+n)\omega}{b(b(2+n) - 2n^2 a_0 p_5)}}x + \rho\right)}. \tag{31}$$

where $\varsigma = x + \frac{2bt^\beta\sqrt{-b(2+n)\omega + 2a_0(bp_1 + n^2\omega p_5)}}{\beta\sqrt{b(b(2+n) - 2n^2 a_0 p_5)}}$. This represents a dark soliton solution.

$$q_7(x,t) = \left(a_0 \pm \frac{\sqrt{\frac{2n^2 a_0^2 p_1(b(2+n) - 2n^2 a_0 p_5)}{2b(2+n) - 4n^2 a_0 p_5}}\coth\left[\sqrt{\frac{n^2 a_0 p_1}{2b(2+n) - 4n^2 a_0 p_5}}(l+\varsigma)\right]}{n\sqrt{p_1}}\right)^{\frac{1}{n}} e^{i\left(\omega\frac{t^\beta}{\beta} \pm \sqrt{\frac{2a_0(bp_1 + n^2\omega p_5) - b(2+n)\omega}{b(b(2+n) - 2n^2 a_0 p_5)}}x + \rho\right)}. \tag{32}$$

$$q_8(x,t) = \left(a_0 \pm \frac{\sqrt{2}\chi_1}{n\sqrt{p_1}}(i\ \mathrm{sech}[2h_1(l+\varsigma)] + \tanh[2h_1(l+\varsigma)])\right)^{\frac{1}{n}} e^{i\left(\omega\frac{t^\beta}{\beta} \pm \sqrt{\frac{2a_0(bp_1 + n^2\omega p_5) - b(2+n)\omega}{b(b(2+n) - 2n^2 a_0 p_5)}}x + \rho\right)}, \tag{33}$$

where $\chi_1 = \frac{1}{n\sqrt{p_1}}\sqrt{\frac{n^2 a_0^2 p_1(b(2+n) - 2n^2 a_0 p_5)}{2b(2+n) - 4n^2 a_0 p_5}}$ and $h_1 = \sqrt{\frac{n^2 a_0 p_1}{2b(2+n) - 4n^2 a_0 p_5}}$. Equation (32) is a singular soliton, while Equation (33) represents a complexiton solution.

$$q_9(x,t) = \left(a_0 \pm \frac{\sqrt{2}\chi_1}{n\sqrt{p_1}}(csch[2h_1(l+\varsigma)] - \coth[2h_1(l+\varsigma)])\right)^{\frac{1}{n}} e^{i\left(\omega\frac{t^\beta}{\beta} \pm \sqrt{\frac{2a_0(bp_1 + n^2\omega p_5) - b(2+n)\omega}{b(b(2+n) - 2n^2 a_0 p_5)}}x + \rho\right)}, \tag{34}$$

and is a singular–singular straddled soliton.

$$q_{10}(x,t) = \left(a_0 \pm \frac{\chi_1}{n\sqrt{2p_1}}\left(\coth\left[\frac{h_1}{2}(l+\varsigma)\right] + \tanh\left[\frac{h_1}{2}(l+\varsigma)\right]\right)\right)^{\frac{1}{n}} e^{i\left(\omega\frac{t^\beta}{\beta} \pm \sqrt{\frac{2a_0(bp_1 + n^2\omega p_5) - b(2+n)\omega}{b(b(2+n) - 2n^2 a_0 p_5)}}x + \rho\right)}. \tag{35}$$

This solution represents a dark–singular straddled soliton.

Result–3:

$$a_1=\frac{\sqrt{-5b(1+n)+22n^2a_0p_5}}{\pm\sqrt{n^2(5p_2+4a_0p_3)}},$$

$$k=\pm\sqrt{\frac{-5b(1+n)w-4ba_0^2(5p_2+4a_0p_3)+22n^2wa_0p_5}{b(5b(1+n)-22n^2a_0p_5)}},$$

$$p_1=\frac{2a_0(5p_2+4a_0p_3)(b(2+n)-2n^2a_0p_5)}{-5b(1+n)+22n^2a_0p_5},$$

$$p_4=\frac{3b(1+n)p_3-6n^2(p_2+3a_0p_3)p_5}{2a_0(-5b(1+n)+22n^2a_0p_5)},$$

$$h_0=\frac{n^2a_0^2(5p_2+4a_0p_3)}{5b(1+n)-22n^2a_0p_5},$$

$$p_6=\frac{-b(1+n)p_3+2n^2(p_2+3a_0p_3)p_5}{4n^2a_0(5p_2+4a_0p_3)}. \tag{36}$$

By substituting the functions $G_j(\varsigma)$ for $j=1,\ldots,5$ into equation (10) and using equations (3), (6), and (7), we can find the solutions to equation (1). This involves inserting Result-3 into $q_j(x,t)$, where $j=10,\ldots,15$, to obtain the following soliton solutions:

$$q_{11}(x,t)=\left(\begin{matrix}a_0\pm\chi_3\sqrt{-\frac{n^2a_0^2(5p_2+4a_0p_3)}{5b(1+n)-22n^2a_0p_5}}\times\\ \tanh\left[\sqrt{-\frac{n^2a_0^2(5p_2+4a_0p_3)}{5b(1+n)-22n^2a_0p_5}}(l+\varsigma)\right]\end{matrix}\right)^{\frac{1}{n}}\times e^{i\left(\omega\frac{t^\beta}{\beta}\pm\sqrt{\frac{-5b(1+n)w-4ba_0^2(5p_2+4a_0p_3)+22n^2wa_0p_5}{b(5b(1+n)-22n^2a_0p_5)}}x+\rho\right)}, \tag{37}$$

where

$$\varsigma=x-\frac{2bt^\beta\sqrt{-5b(1+n)\omega-4ba_0^2(5p_2+4a_0p_3)+22n^2\omega a_0p_5}}{\beta\sqrt{b(5b(1+n)-22n^2a_0p_5)}}$$

and $\chi_3=\frac{\sqrt{-5b(1+n)+22n^2a_0p_5}}{\sqrt{n^2(5p_2+4a_0p_3)}}$. This solution represents a dark soliton.

$$q_{12}(x,t)=\left(\begin{matrix}a_0\pm\chi_3\sqrt{-\frac{n^2a_0^2(5p_2+4a_0p_3)}{5b(1+n)-22n^2a_0p_5}}\times\\ \coth\left[\sqrt{-\frac{n^2a_0^2(5p_2+4a_0p_3)}{5b(1+n)-22n^2a_0p_5}}(l+\varsigma)\right]\end{matrix}\right)^{\frac{1}{n}}\times e^{i\left(\omega\frac{t^\beta}{\beta}\pm\sqrt{\frac{-5b(1+n)w-4ba_0^2(5p_2+4a_0p_3)+22n^2wa_0p_5}{b(5b(1+n)-22n^2a_0p_5)}}x+\rho\right)}, \tag{38}$$

which is a singular soliton.

$$q_{13}(x,t)=\left(a_0\pm\chi_3h_2(i\,\mathrm{sech}[2h_2(l+\varsigma)]+\tanh[2h_2(l+\varsigma)])\right)^{\frac{1}{n}}\times e^{i\left(\omega\frac{t^\beta}{\beta}\pm\sqrt{\frac{-5b(1+n)w-4ba_0^2(5p_2+4a_0p_3)+22n^2wa_0p_5}{b(5b(1+n)-22n^2a_0p_5)}}x+\rho\right)}, \tag{39}$$

where $h_2=\sqrt{-\frac{n^2a_0^2(5p_2+4a_0p_3)}{5b(1+n)-22n^2a_0p_5}}$. This is a complexiton solution.

$$q_{14}(x,t)=\left(a_0\pm\chi_3h_2(csch[2h_2(l+\varsigma)]-\coth[2h_2(l+\varsigma)])\right)^{\frac{1}{n}}\times e^{i\left(\omega\frac{t^\beta}{\beta}\pm\sqrt{\frac{-5b(1+n)w-4ba_0^2(5p_2+4a_0p_3)+22n^2wa_0p_5}{b(5b(1+n)-22n^2a_0p_5)}}x+\rho\right)}, \tag{40}$$

and it is a singular–singular straddled soliton.

$$q_{15}(x,t)=\left(a_0\pm\frac{\chi_3h_2}{2}\left(\coth\left[\frac{h_2}{2}(l+\varsigma)\right]+\tanh\left[\frac{h_2}{2}(l+\varsigma)\right]\right)\right)^{\frac{1}{n}}\times e^{i\left(\omega\frac{t^\beta}{\beta}\pm\sqrt{\frac{-5b(1+n)w-4ba_0^2(5p_2+4a_0p_3)+22n^2wa_0p_5}{b(5b(1+n)-22n^2a_0p_5)}}x+\rho\right)}, \tag{41}$$

which is a singular-dark straddled soliton.

4. Conclusions

The current paper studied the governing nonlinear Schrödinger equation with fractional temporal evolution and linear chromatic dispersion with Kudryashov's extended-exotic form of SPM. The integration algorithm adopted in the work is the simplest equation approach and this turned out to be an epic failure to retrieve bright optical soliton solutions to the model. Instead, other forms of soliton solutions, straddled solitons and complexiton solutions are revealed. As the title of the work indicates, the internet bottleneck effect is not controlled. Therefore, the stern message is that the integration algorithm, namely the simplest equation approach, is not robust to retrieve the most important form of soliton solutions, namely the bright solitons.

While the results of the paper are discouraging the message is that additional integration approaches would be later identified to successfully recover the much-demanding bright soliton solutions to this governing model. The recovered results from those additional integration algorithms will be made available across the board. Once these recovered results are available and aligned with the pre-existing ones, they will be disseminated.

Disclosure

The authors claim there is no conflict of interest.

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Подолання вузьких місць в мережі Інтернет за допомогою розширеної та екзотичної форми Кудряшова структури фазової самомодуляції з дробовою часовою еволюцію

M.A.S. Murad, A.H. Arnous, Y. Yildirim & A. Biswas

Анотація. Ця стаття оптимістично та агресивно спрямована на контроль ефекту вузького місця в мережі Інтернет за допомогою розширеної та екзотичної форми Кудряшова структури фазової самомодуляції з використанням модифікованого методу простого рівняння. Цей підхід, на жаль, не дав можливість отримати роз'язки для випадку яскравих солітонів, які служать бітами при передачі інформації. Дана модель передбачає дробову часову еволюцію. Неспроможність схеми інтеграції отримати роз'язки для випадку яскравих солітонів дає в цій статті суворий урок щодо прийнятого ненадійного алгоритму інтеграції.

Ключові слова: солітони, метод Кудряшова, дробова часова еволюція.