

Cubic-quartic optical soliton perturbation with Fokas–Lenells equation having Kerr law of self-phase modulation by Laplace–Adomian decomposition

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Abstract. This paper addresses the perturbed Fokas–Lenells equation with a cubic-quartic form of dispersive effects and the Kerr law of self-phase modulation structures. The chromatic dispersive effect is replaced with a collective count of third- and fourth-order dispersions when the count on CD runs low. The Laplace–Adomian decomposition scheme made this numerical study possible. Both bright and dark optical solitons are studied using this principle. The error count is impressively low, thus making this numerical approach a viable one.

Keywords: Fokas–Lenells equation, Laplace–Adomian decomposition, cubic-quartic optical solitons.

<https://doi.org/10.15407/spqeo29.02.203>

PACS 42.81.Gs, 42.65.Jx, 42.65.Tg

Manuscript received 10.09.25; revised version received 24.04.26; accepted for publication 10.06.26; published online 23.06.26.

1. Introduction

The concept of cubic-quartic optical solitons emerged about a decade or so ago. The necessity for this concept comes from the fact that occasionally the geometry of an optical fiber and its rough handling and other physical reasons could lead to a low count of the much-needed chromatic dispersion (CD). Thus, to replenish the necessary balance between dispersion and self-phase modulation (SPM), it is imperative to replace CD with a collective count of third and fourth-order dispersions. This would lead to the sustainment of the balance and thus ensure the stable propagation of such solitons through the optical fibers for intercontinental distances.

The governing model that would be studied in this paper is the Fokas–Lenells equation, which is an extension of the familiar nonlinear Schrodinger’s equation. The extension is achieved by the inclusion of the soliton self-frequency shift. Additionally, a couple of Hamiltonian-type perturbative effects are also taken into account that would bring about performance enhancement in the soliton transmission dynamics through optical fibers. Thus, the perturbed Fokas–Lenells equation is the equation of study in this paper. The approach is through the Laplace–Adomian decomposition method (LADM). Both bright and dark soliton solutions are examined numerically using this integration scheme. The surface plots are presented along

with the error analysis. The details are penned down after a com-prehensive introduction to the numerical integration scheme.

2. The model for the perturbed cubic-quartic Fokas-Lenells equation

The dimensionless perturbed expression for the cubic-quartic Fokas-Lenells equation (CQ-FLE), including both third-order dispersion (3OD) and fourth-order dispersion (4OD) terms, as instead of only the conventional dispersion terms considered in the FLE with Kerr nonlinearity, is presented as follows [1]:

$$iq_t + iaq_{xxx} + bq_{xxxx} + |q|^2(cq + idq_x) = i[\lambda(|q|^2q)_x + \mu(|q|^2)_x q]. \quad (1)$$

This equation was initially investigated in [2–10] and is present in several systems, including fluid dynamics, solid-state physics, chemical reactions, condensed matter, nonlinear optics, and plasma physics. The function $q(x,t)$ represents a complex field envelope in Eq. (1), where x and t denote spatial and temporal variables, respectively. In this context, a and b represent the third and fourth-order dispersion coefficients, respectively. Furthermore, self-phase modulation is denoted by the coefficient c , whereas d represents soliton self-frequency shift. From the perturbation terms, λ represents the self-steepening effect while μ accounts for the other form of soliton self-frequency shift.

2.1. Solutions in relation to bright optical solitons

The form of the solution to Eq. (1) for bright optical solitons is provided in [1, 6] by

$$q(x, t) = A \operatorname{sech}^2[B(x - vt)] e^{i[-\kappa x + \omega t + \Omega]}. \quad (2)$$

The soliton velocity is denoted with v , the soliton frequency – by ω , the angular velocity – by κ , and the phase center – as Ω .

Inside this framework, the inverse width B is given by

$$B = \left[\frac{9\kappa(a - 2b\kappa) \sqrt{81\kappa^2(a - 2b\kappa)^2 + 300b(\omega + a\kappa^3 - b\kappa^4)}}{200b} \right]^{\frac{1}{2}} \quad (3)$$

with the restrictions:

$$27\kappa^2(a - 2b\kappa)^2 + 100b(\omega + a\kappa^3 - b\kappa^4) \geq 0, \quad (4)$$

and

$$b\sqrt{81\kappa^2(a - 2b\kappa)^2 + 300b(\omega + a\kappa^3 - b\kappa^4)} > 0. \quad (5)$$

The amplitude A of the soliton in terms of its width B is given by

$$A = \left[\frac{B^2(21(a - 2b\kappa) - 40bB^2)}{12(c + (d - \lambda)k)} \right]^{\frac{1}{2}}, \quad (6)$$

with the restriction:

$$(c + (d - \lambda)k)(21(a - 2b\kappa) - 40bB^2) > 0. \quad (7)$$

The relationship between some of the coefficients of Eq. (1) and the soliton speed is

$$v = 4b\kappa^3 - 3a\kappa^2. \quad (8)$$

In the above context, κ refers to any parameter satisfying Eq. (8).

2.2. Solutions in relation to dark optical solitons

The solution to Eq. (1) for dark optical solitons is provided in [6] by

$$u(x, t) = A\{B + 2\tanh^2[(x - vt)]\}e^{i[-\kappa x + \omega t + \Omega]}. \quad (9)$$

The soliton velocity is denoted with v , the soliton frequency – by ω , the angular velocity – by κ , and the phase center – as Ω .

The parameters A and B are expressed in terms of the coefficients of Eq. (1) as follows:

$$A = \sqrt{-\frac{30b}{d\kappa - \lambda\kappa + c}}, \quad B = \frac{3\kappa^2 - 20}{15}. \quad (10)$$

Soliton velocity depends on the fourth-order dispersion coefficient and angular velocity:

$$v = 8b\kappa^3. \quad (11)$$

The soliton frequency ω is also related to the coefficients of the model to be studied by means of the relationship

$$\omega = b\left(\frac{16}{3} - 22\kappa^2\right) \quad (12)$$

and the restrictions on the parameters are as follows

$$9\kappa^4 - 30\kappa^2 - 40 = 0, \quad d - 3\lambda - 2\mu = 0, \quad a = 4\kappa b, \quad (13)$$

$$b(c + d\kappa - \lambda\kappa) < 0, \quad (14)$$

κ is any real constant satisfying Eqs. (13) and (14).

3. An overview of the Laplace-Adomian decomposition method

G. Adomian proposed the Adomian decomposition method in the 1980’s and used it to many stochastic and deterministic problems in science and engineering [11]. The solution is derived from an ordered series, together with the decomposition of the nonlinear operator into a series whose terms are recursively calculated using the known Adomian polynomials. The notion of integrating the decomposition approach with the fundamental Laplace transform, so formulating the Adomian-Laplace decomposition method, was originally introduced by G. Adomian and R. Rach in [12].

To better comprehend the key elements of the Laplace-Adomian decomposition method, we first examine the traditional format of a partial differential equation with nonlinear components, operationally denoted by $F(q(x, t)) = 0$, with relation to the initial condition

$$q(x, 0) = f(x). \quad (15)$$

The symbol F represents a differential operator. Now let us disintegrate the operator F into constituent parts $F = D + R + N$. The expression $D(q) = \frac{\partial q}{\partial t}$ denotes a linear differential operator. The remaining linear and nonlinear components are denoted by the operators R and N , respectively.

With these issues, Eq. (15) may be rewritten as

$$Dq(x, t) = Rq(x, t) + Nq(x, t). \quad (16)$$

Solving for $Dq(x, t)$ and then using the well-known Laplace transform to Eq. (16), results

$$\mathcal{L}\{Dq(x, t)\} = \mathcal{L}\{Rq(x, t) + Nq(x, t)\}. \quad (17)$$

Thus, Eq. (17)) is equivalent to

$$sq(x, s) = q(x, 0) + \mathcal{L}\{Rq(x, t) + Nq(x, t)\}. \quad (18)$$

By using initial condition, one obtains

$$q(x, s) = \frac{f(x)}{s} + \frac{1}{s} \mathcal{L}\{Rq(x, t) + Nq(x, t)\}. \quad (19)$$

Applying now the inverse operator \mathcal{L}^{-1} of \mathcal{L} , to each sides of Eq. (19) produces the following result:

$$q(x, t) = f(x) + \mathcal{L}^{-1}\left[\frac{1}{s} \mathcal{L}\{Rq(x, t) + Nq(x, t)\}\right]. \quad (20)$$

The Laplace–Adomian decomposition approach is founded on the notion that the solution $q(x, t)$ may be expressed as a function series:

$$q(x, t) = \sum_{n=0}^{\infty} q_n(x, t). \quad (21)$$

Additionally, the nonlinear term N decomposes as a result of the Adomian technique as

$$Nq(x, t) = \sum_{n=0}^{\infty} A_n(q_0, q_1, \dots, q_n). \quad (22)$$

Each A_n is an Adomian polynomial involving variables q_0, q_1, \dots, q_n that may be computed for every type of non-linearity in compliance with the recurrent formula [13–15]:

$$\begin{cases} A_0 = N(q_0) & n = 0, \\ A_n = \frac{1}{n} \sum_{k=0}^{n-1} (k+1)q_{k+1} \frac{\partial}{\partial q_0} A_{n-1-k} & n \geq 1. \end{cases} \quad (23)$$

Therefore, the first Adomian’s polynomials using recursive formula (23), are given by

$$A_0 = N(q_0)$$

$$A_1 = q_1 N'(q_0)$$

$$A_2 = q_2 N'(q_0) + \frac{1}{2} q_1^2 N''(q_0)$$

$$A_3 = q_3 N'(q_0) + q_1 q_2 N''(q_0) + \frac{1}{3!} q_1^3 N^{(3)}(q_0)$$

$$A_4 = q_4 N'(q_0) + \left(\frac{1}{2} q_2^2 + q_1 q_3\right) N''(q_0) + \frac{1}{2!} q_1^2 q_2 N^{(3)}(q_0) + \frac{1}{4!} q_1^4 N^{(4)}(q_0)$$

$$A_5 = q_5 N'(q_0) + (q_1 q_4 + q_2 q_3) N''(q_0) + \frac{1}{2!} (q_3 q_1^2 + q_1 q_2^2) N^{(3)}(q_0) + \frac{1}{3!} q_1^3 q_2 N^{(4)}(q_0) + \frac{1}{5!} q_1^5 N^{(5)}(q_0).$$

All other polynomials are computed in the same manner.

The formula (23) has the advantage that it does not require tedious calculations involving differentiation with respect to all the variables since only for its application it is necessary to do arithmetic sums and differentiate only with respect to u_0 .

Substituting (21) and (22) into Eq. (20) gives rise to

$$\sum_{n=0}^{\infty} q_n(x, t) = f(x) + \mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \left\{ R \sum_{n=0}^{\infty} q_n(x, t) + \sum_{n=0}^{\infty} A_n(q_0, q_1, \dots, q_n) \right\} \right]. \quad (24)$$

As a result, Eq. (24) suggests an iterative procedure given by

$$\begin{cases} q_0(x, t) = f(x), \\ q_{n+1}(x, t) = \mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L} \{ R q_n(x, t) + A_n(q_0, q_1, \dots, q_n) \} \right], \\ n = 0, 1, 2, \dots \end{cases} \quad (25)$$

Collecting each of u_n produced by way of (25) allows us to approximate the solution of Eq. (1) to N -summands by means of

$$q_N(x, t) = \sum_{n=0}^{N-1} q_n(x, t), \quad N \geq 1. \quad (26)$$

This study demonstrated that using the Adomian decomposition technique in conjunction with the Laplace transform involves significantly less computational work than using the classic Adomian decomposition approach. The suggested approach significantly reduces the amount of computations required. Additionally, the Laplace–Adomian decomposition method (LADM) can significantly reduce the computational effort compared to other methods. The provided algorithm is simple to implement and does not involve linearization or discretization.

4. A Fokas–Lenells equation (1) algorithm built from the presented technique

In this part, we will illustrate the applicability of LADM to find any particular solution to Eq. (1) that satisfies the condition at $t = 0$: $q(x, 0) = f(x)$.

To apply the LADM method to Eq. (1), we will formulate it in terms of operators as

$$Dq(x, t) + Rq(x, t) + (N_1 + N_2 + N_3)q(x, t) = 0. \quad (27)$$

In Eq. (27), $N_1, N_2,$ and N_3 represent the nonlinear operators acting on q : $-i|q|^2(cq + idq_x)$, $-\lambda(|q|^2 q)_x$, and $-\mu q(|q|^2)_x$, respectively. While $Rq = aq_{xxx} - ibq_{xxxx}$ represents the linear differential operator, $Dq = q_t$ simply signifies time derivative.

The LADM expresses solution q as an infinite series whose summands are functions, and it is represented as follows:

$$q(x, t) = \sum_{n=0}^{\infty} q_n(x, t). \quad (28)$$

$N_1, N_2,$ and N_3 are nonlinear terms operating on the function q and can be decomposed into an infinite sequence of Adomian polynomials denoted by the expressions:

$$N_1 q = -i|q|^2(cq + idq_x) = \sum_{n=0}^{\infty} P_n(q_0, q_1, \dots, q_n), \quad (29)$$

$$N_2 q = -\lambda(|q|^2 q)_x = \sum_{n=0}^{\infty} Q_n(q_0, q_1, \dots, q_n), \quad (30)$$

and

$$N_3 q = -\mu q(|q|^2)_x = \sum_{n=0}^{\infty} S_n(q_0, q_1, \dots, q_n). \quad (31)$$

In expressions (29)–(31), $P_n, Q_n,$ and S_n are the polynomials known as the Adomian polynomials, which may be determined using the method provided by Eq. (23), *i.e.*

$$P_0 = N_1(q_0), \quad Q_0 = N_2(q_0), \quad S_0 = N_3(q_0),$$

and for every $n \geq 1$ we have

$$P_n = \frac{1}{n} \sum_{k=0}^{n-1} (k+1)q_{k+1} \frac{\partial}{\partial q_0} P_{n-1-k},$$

$$Q_n = \frac{1}{n} \sum_{k=0}^{n-1} (k+1)q_{k+1} \frac{\partial}{\partial q_0} Q_{n-1-k} \text{ and} \quad (32)$$

$$S_n = \frac{1}{n} \sum_{k=0}^{n-1} (k+1)q_{k+1} \frac{\partial}{\partial q_0} S_{n-1-k}.$$

Finally, we denote by A_n the Adomian polynomials resulting from the nonlinear operator $N_1 + N_2 + N_3$ for every $n \geq 0$.

Then, the first Adomian polynomials that correspond to the nonlinear operator $N_1 + N_2 + N_3$ are given as

$$A_0 = -icq_0^2 \bar{q}_0 - (\lambda + \mu)q_0^2 \bar{q}_{0x} + (d - 2\lambda - \mu)q_0 \bar{q}_0 q_{0x},$$

$$A_1 = -2icq_0 q_1 \bar{q}_0 - icq_0^2 \bar{q}_1 - (\lambda + \mu) \times (q_0^2 \bar{q}_{1x} + 2q_0 q_1 \bar{q}_{0x}) + (d - 2\lambda - \mu) \times (q_0 \bar{q}_0 q_{1x} + q_0 \bar{q}_1 q_{0x} + q_1 \bar{q}_0 q_{0x}),$$

$$A_2 = -2icq_0 q_2 \bar{q}_0 - icq_1^2 \bar{q}_0 - 2icq_0 q_1 \bar{q}_1 - icq_0^2 \bar{q}_2 - (\lambda + \mu)(q_1^2 \bar{q}_{0x} + q_0^2 \bar{q}_{2x} + 2q_0 q_1 \bar{q}_{1x} + 2q_0 q_2 \bar{q}_{0x}) + (d - 2\lambda - \mu)(q_0 \bar{q}_0 q_{2x} + q_0 \bar{q}_1 q_{1x} + q_0 \bar{q}_2 q_{0x} + q_1 \bar{q}_0 q_{1x} + q_1 \bar{q}_1 q_{0x} + q_2 \bar{q}_0 q_{0x}),$$

$$A_3 = -2icq_0 q_3 \bar{q}_0 - 2icq_1 q_2 \bar{q}_0 - 2icq_0 q_2 \bar{q}_1 - icq_1^2 \bar{q}_1 - 2icq_0 q_1 \bar{q}_2 - icq_0^2 \bar{q}_3 - (\lambda + \mu) \times (q_1^2 \bar{q}_{1x} + q_0^2 \bar{q}_{3x} + 2q_0 q_1 \bar{q}_{2x} + 2q_0 q_2 \bar{q}_{1x} + 2q_0 q_3 \bar{q}_{0x} + 2q_1 q_2 \bar{q}_{0x}) + (d - 2\lambda - \mu) \times (q_0 \bar{q}_0 q_{3x} + q_0 \bar{q}_1 q_{2x} + q_0 \bar{q}_2 q_{1x} + q_0 \bar{q}_3 q_{0x} + q_1 \bar{q}_0 q_{2x} + q_1 \bar{q}_1 q_{1x} + q_1 \bar{q}_2 q_{0x} + q_2 \bar{q}_0 q_{1x} + q_2 \bar{q}_1 q_{0x} + q_3 \bar{q}_0 q_{0x}).$$

$$A_4 = -ic\bar{q}_0 q_2^2 - 2icq_0 \bar{q}_0 q_4 - 2ic\bar{q}_0 q_1 q_3 - 2icq_0 \bar{q}_1 q_3 - 2icq_1 \bar{q}_1 q_2 + 2c\bar{q}_0 \bar{q}_2 q_2 - icq_1^2 \bar{q}_2 - 2icq_0 \bar{q}_1 q_3 - icq_0^2 \bar{q}_4 - (\lambda + \mu)(q_2^2 \bar{q}_{0x} + q_1^2 \bar{q}_{2x} + 2q_0 q_1 \bar{q}_{3x} + 2q_0 q_2 \bar{q}_{2x} + 2q_0 q_3 \bar{q}_{1x} + 2q_0 q_4 \bar{q}_{0x} + 2q_1 q_2 \bar{q}_{1x} + 2q_1 q_3 \bar{q}_{0x}) + (d - 2\lambda - \mu)(q_0 \bar{q}_0 q_{4x} + q_0 \bar{q}_1 q_{3x} + q_0 \bar{q}_2 q_{2x} + q_0 \bar{q}_3 q_{1x} + q_0 \bar{q}_4 q_{0x} + q_1 \bar{q}_0 q_{3x} + q_1 \bar{q}_1 q_{2x} + q_1 \bar{q}_2 q_{1x} + q_1 \bar{q}_3 q_{0x} + q_2 \bar{q}_0 q_{2x} + q_2 \bar{q}_1 q_{1x} + q_2 \bar{q}_2 q_{0x} + q_3 \bar{q}_0 q_{1x} + q_3 \bar{q}_1 q_{0x} + q_4 \bar{q}_0 q_{0x}),$$

Similarly, the remainder of the polynomials is produced in a similar way.

By implementing the Laplace integral transform with respect to t on both parts of Eq. (27) and using its linearity, the following result is obtained:

$$\mathcal{L}\{Dq(x, t)\} = -\mathcal{L}\{Rq(x, t)\} - \mathcal{L}\{(N_1 + N_2 + N_3)q(x, t)\}. \quad (33)$$

Due to the differentiation characteristic of the Laplace transform, (33) may be expressed as

$$s\mathcal{L}\{q(x, t)\} - q(x, 0) = -\mathcal{L}\{Rq(x, t)\} - \mathcal{L}\{(N_1 + N_2 + N_3)q(x, t)\}. \quad (34)$$

Rewriting the preceding expression,

$$\mathcal{L}\{q(x, t)\} = \frac{1}{s} q(x, 0) - \frac{1}{s} (\mathcal{L}\{Rq(x, t)\} - \mathcal{L}\{(N_1 + N_2 + N_3)q(x, t)\}). \quad (35)$$

By substituting (28), (29), (30) and (31) into (35), we acquire

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} q_n(x, t)\right\} = \frac{f(x)}{s} - \frac{1}{s} \left(\mathcal{L}\left\{R \sum_{n=0}^{\infty} q_n(x, t)\right\} + \mathcal{L}\left\{\sum_{n=0}^{\infty} P_n\right\} + \mathcal{L}\left\{\sum_{n=0}^{\infty} Q_n\right\} + \mathcal{L}\left\{\sum_{n=0}^{\infty} S_n\right\} \right). \quad (36)$$

Comparing algebraically both sides of Eq. (36), the following relationships emerge:

$$\mathcal{L}\{q_0(x, t)\} = \frac{f(x)}{s}. \quad (37)$$

In general, we deduct recursively:

$$\mathcal{L}\{q_{n+1}(x, t)\} = -\frac{1}{s} (\mathcal{L}\{Rq_n(x, t)\} + \mathcal{L}\{P_n + Q_n + S_n\}) = -\frac{1}{s} (\mathcal{L}\{Rq_n(x, t)\} + \mathcal{L}\{A_n\}), \quad n \geq 1.$$

The following recurrence formulae are deduced for each $n = 0, 1, 2, \dots$ by employing the inverse Laplace transformation

$$\begin{cases} q_0(x, t) = f(x), \\ q_{n+1}(x, t) = -\mathcal{L}^{-1} \left[\frac{1}{s} \mathcal{L}\{Rq_n(x, t) + A_n(q_0, \dots, q_n)\} \right]. \end{cases} \quad (38)$$

Table 1. Coefficients in Eq. (1) utilized for the simulation of bright solitons.

Examples	a	b	c	d	λ	μ	N	Max Error
1	1.40	-0.65	6.20	-20.5	-8.20	2.05	12	$8.0 \cdot 10^{-9}$
2	1.55	3.26	2.90	-18.6	-9.24	4.53	12	$6.0 \cdot 10^{-0}$
3	0.55	0.16	0.90	-14.9	-8.55	5.33	12	$1.0 \cdot 10^{-8}$

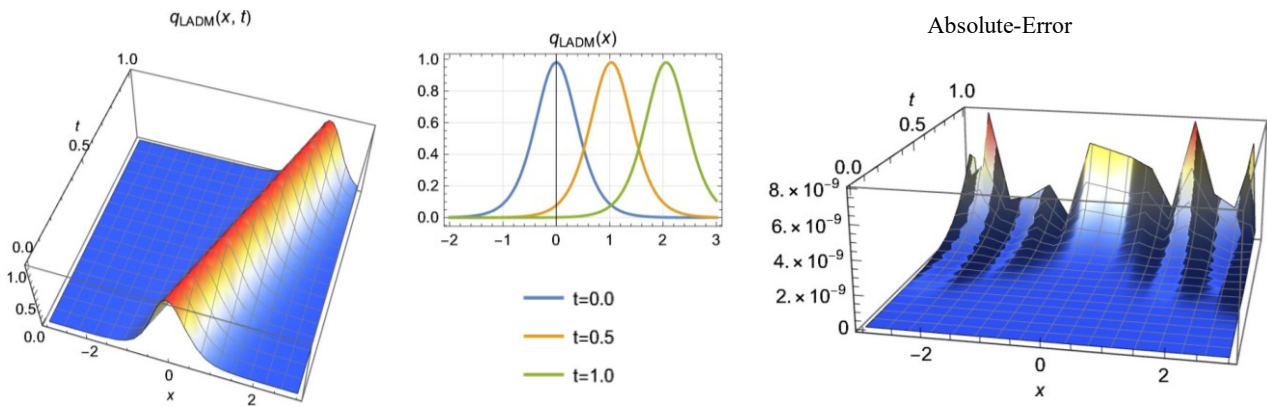


Fig. 1. Bright soliton profile $q_{LADM}(x,t)$ in the interval $-3.0 \leq x \leq 3.0$ for the parameters selected in example 1 (left). 2D graph for $t = 0.0, t = 0.5$ and $t = 1.0$ (center). Absolute error by LADM with $N = 12$ (right).

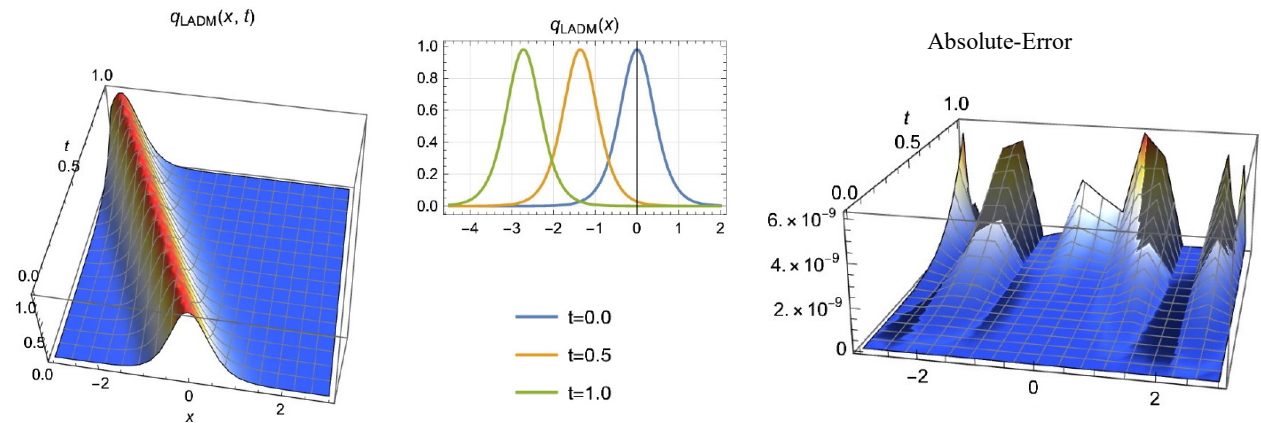


Fig. 2. Bright soliton profile $q_{LADM}(x,t)$ in the interval $-3.0 \leq x \leq 3.0$ for the parameters selected in example 2 (left). 2D graph for $t = 0.0, t = 0.5$ and $t = 1.0$ (center). Absolute error by LADM with $N = 12$ (right).

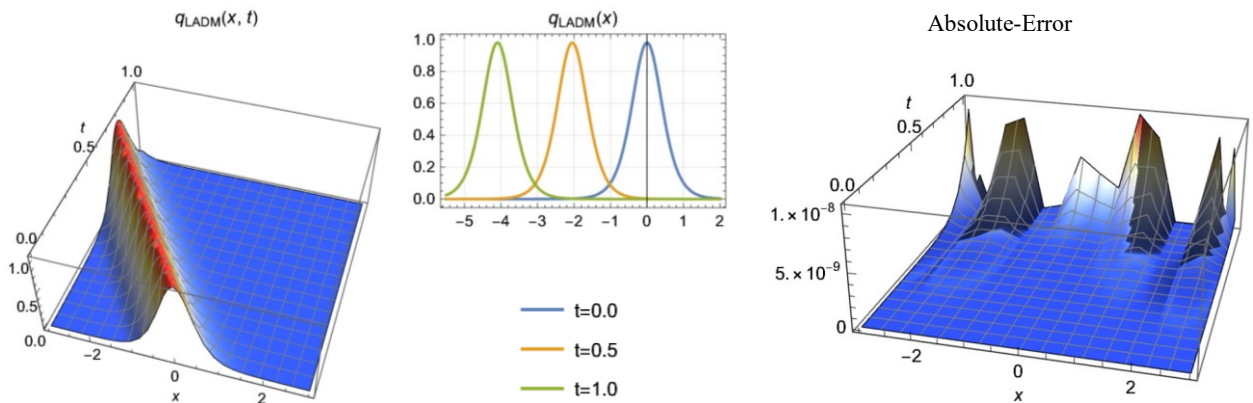


Fig. 3. Bright soliton profile $q_{LADM}(x,t)$ in the interval $-3.0 \leq x \leq 3.0$ for the parameters selected in example 3 (left). 2D graph for $t = 0.0, t = 0.5$ and $t = 1.0$ (center). Absolute error by LADM with $N = 12$ (right).

Table 2. Coefficients in Eq. (1) utilized for the simulation of dark solitons.

Examples	a	b	c	d	λ	μ	N	[Max Error]
4	0.70	3.25	0.80	0.44	-3.25	5.50	12	$4.0 \cdot 10^{-9}$
5	4.05	-1.77	1.60	-4.15	-4.55	4.75	12	$1.0 \cdot 10^{-8}$
6	2.50	-2.75	0.60	-8.55	-7.15	6.45	12	$6.0 \cdot 10^{-9}$

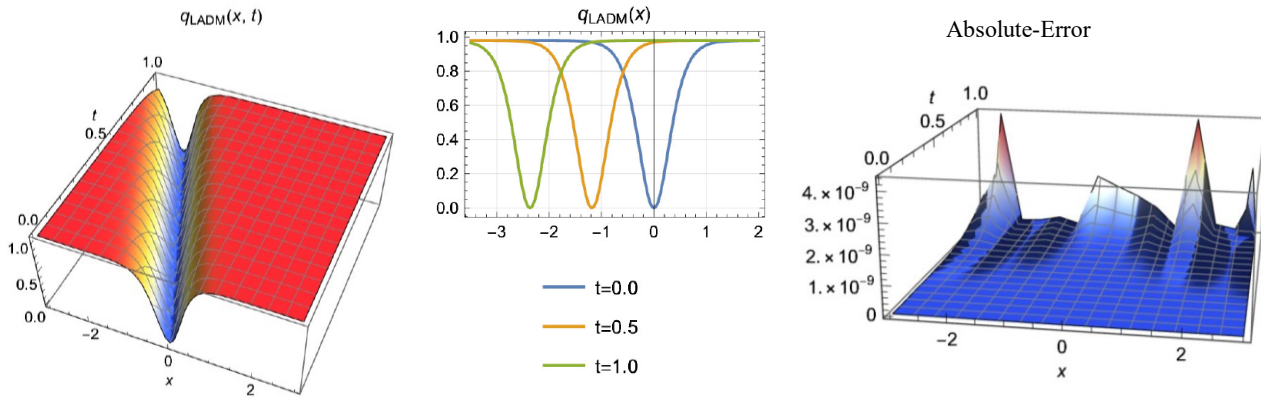


Fig. 4. Dark soliton profile $q_{LADM}(x,t)$ in the interval $-3.0 \leq x \leq 3.0$ for the parameters selected in example 4 (left). 2D graph for $t = 0.0, t = 0.5$ and $t = 1.0$ (center). Absolute error by LADM with $N = 12$ (right).

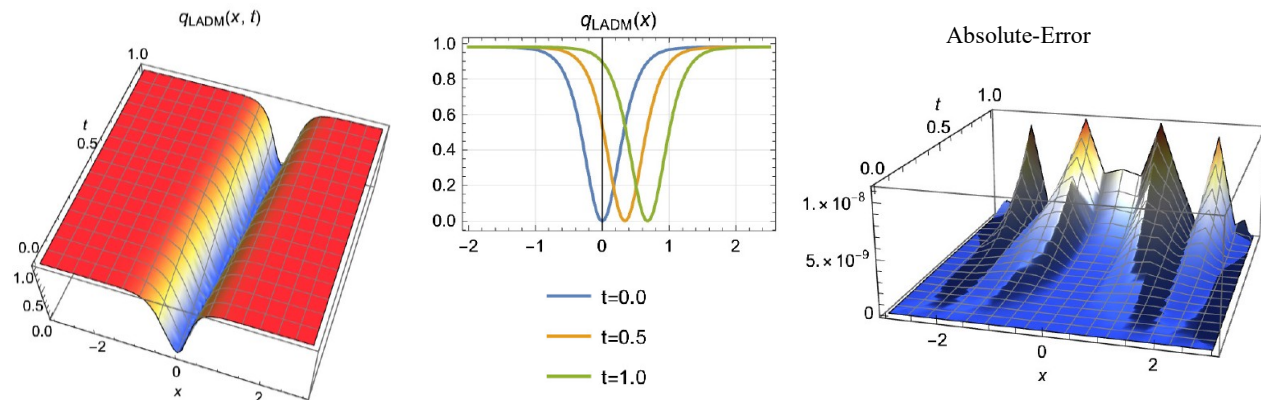


Fig. 5. Dark soliton profile $q_{LADM}(x,t)$ in the interval $-3.0 \leq x \leq 3.0$ for the parameters selected in example 5 (left). 2D graph for $t = 0.0, t = 0.5$ and $t = 1.0$ (center). Absolute error by LADM with $N = 12$ (right).

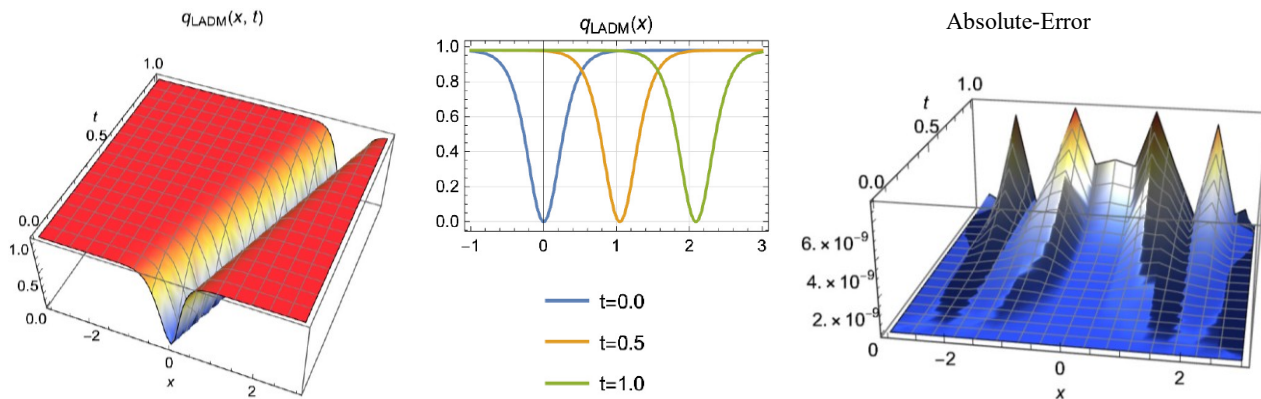


Fig. 6. Dark soliton profile $q_{LADM}(x,t)$ in the interval $-3.0 \leq x \leq 3.0$ for the parameters selected in example 6 (left). 2D graph for $t = 0.0, t = 0.5$ and $t = 1.0$ (center). Absolute error by LADM with $N = 12$ (right).

5. Displaying examples and graphic solutions

This section will employ the LADM-derived algorithm described by Eq. (38) to simulate both bright and dark solitons for Eq. (1).

5.1. Simulations of bright optical solitons

We now conduct the simulation of the three examples mentioned in Table 1, with their results and absolute errors presented in Figs. 1, 2, and 3. These figures illustrate the performance of the simulation, highlighting that there are no discrepancies between the predicted and actual values. The absolute errors offer valuable insights into the accuracy of each example, facilitating a comprehensive analysis of the method's effectiveness.

5.2. Simulations of dark optical solitons

We now conduct the simulation of the three examples mentioned in Table 2, with their results and absolute errors presented in Figs. 4, 5, and 6. These figures illustrate the performance of the simulation, highlighting that there are no discrepancies between the predicted and actual values. The absolute errors offer valuable insights into the accuracy of each example, facilitating a comprehensive analysis of the method's effectiveness.

6. Conclusions

This paper addressed the perturbed cubic-quartic Fokas–Lenells equation by the LADM. Both bright and dark soliton solutions are examined. The error measure that has been recovered is impressively low. This indicates that the numerical scheme adopted in this paper to study the model is efficient and reliable. The numerical scheme is thus reliable to apply to additional optoelectronic devices in the future. A few such devices are fibers with differential group delay, dispersion-flattened fibers, optical couplers, magneto-optic waveguides, Bragg gratings, and several others. The results will be disseminated once available.

Acknowledgment

The corresponding author (AB) is thankful to Grambling State University for the financial support he received as the Endowed Chair of Mathematics.

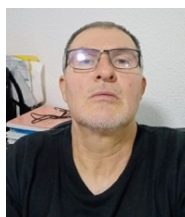
Disclosure

The authors claim there is no conflict of interest.

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Yildirim Y.: validation, writing, review and editing.

Bălănică C.D.: methodology, formal analysis.

de Chávez J.R.: funding acquisition, formal analysis, resources, data curation.

Збурення кубічно-квартичного оптичного солітону в межах рівняння Фокаса–Ленеллса з керрівською фазовою самомодуляцією методом розкладу Лапласа–Адомяна

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Анотація. У цій статті розглядається збурене рівняння Фокаса–Ленеллса з кубічно-квартичною формою дисперсійних ефектів та законом Керра для структур фазової самомодуляції. Хроматичний дисперсійний (CD) ефект замінюється колективним підрахунком дисперсій третього та четвертого порядку, коли внесок CD зменшується. Схема розкладання Лапласа–Адомяна зробила можливим це чисельне дослідження. За цим принципом вивчаються як яскраві, так і темні оптичні солітони. Кількість помилок вражає низька, що робить цей чисельний підхід придатним до використання.

Ключові слова: рівняння Фокаса–Ленеллса, розкладання Лапласа–Адомяна, кубічно-квартичні оптичні солітони.