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Energy processes in combined power supplies with linear capacitors and supercapacitors

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Abstract. In this paper, energy processes in combined power supplies with supercapacitors (nonlinear capacitors) and linear capacitors at their charge from a non-ideal DC source, for which there is a rechargeable battery, have been analyzed. The energy characteristics of charge circuits in combined power supplies with supercapacitors and linear capacitors have been obtained. An analysis of the dependence of the energy difference accumulated in the supercapacitor and in the linear capacitor in the process of charge (from zero initial conditions to voltage $U_f = U_n /n$), on the variations of the parameter n, has been performed. The dependence of the energy ratio accumulated in the capacitors during the charge period on the value of the final voltage on the terminals of the capacitors has been investigated.

Keywords: energy processes, supercapacitor, linear capacitor, combined power supply.

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1. Introduction

In today's trends, conventional linear capacitors (LC) that satisfy high pulse power requirements, which other sources of energy cannot achieve, are predominantly used to provide pulse currents in electronics and electrical engineering. Accumulator batteries (AB) are very popular due to high energy performance and long-term energy-to-load capability. Various power characteristics of capacitors and rechargeable batteries are the basis for research on the development of combined energy sources. In these sources, capacitors are used to provide significant pulsed capacities and currents, and AB – for the purpose of long-term power supply to consumers with constant current. These combined energy sources have not developed for a long time due to low specific energy of LC.

Thanks to the productive scientific developments to improve the new type of energy storage devices – supercapacitors (SC), they are increasingly used in combined systems. The batteries of SC at the present stage have a capacity of thousands of farads with voltages from hundreds to thousands of volts. Fairly high figures for specific capacities of SC are achieved due to the internal resistance of the element smaller than 0.1 mOhm [1–4]. SC can operate at least 1 million cycles without degradation of electrical characteristics, which significantly exceeds the resources of well-known industrial AB designs. The charge voltage of individual elements of SC has a limiting value, after which their irreversible rapid electrochemical decomposition arises. To increase the permissible operation voltage in real systems, separate elements of SC are usually connected in series forming batteries from energy storage devices [5–8].

The main advantage of SC is its specific power, which is more than twenty times exceeds the characteristics of AB. At the same time, the period of charge and discharge of SC is 100–1000-fold less than that of AB. Modern samples of SC have the specific energy seven times lower than that in lithium-ionic AB, while hundreds of times higher than the specific energy in the LC. Based on these features of SC, AB and LC, it is advisable to provide the established modes of electricity consumption by the load from AB, and at pulse consumption of significant power – to work with SC or LC [2, 9–11].

The scientific analysis of the energy characteristics of SC, LC and AB in the combined power supply systems showed [6–11] that the studies were performed without analyzing the dependences of the difference in energy accumulated in SC and LC in the process of charge (from zero initial voltage to $U_f = U_n /n$) on the variation parameter n. The dependence of the energy accumulated in capacitors during the charge time on the value of final voltage on the terminals of capacitors when charging from AB was not investigated.

The purpose of this work is to identify and compare the energy and energy differences that are accumulated in SC and LC, when they are charged from zero initial conditions to a certain final voltage on the terminals $U_f = U_n/n$ and when being charged from non-zero initial conditions up to the nominal voltage on the terminals of SC and LC, depending on the parameter n and on the values of initial U_i and final U_f voltages on the terminals.

2. Theory and experiment

From the scientific papers [4, 10, 12], it is known that the part of the capacity of SC depends on the voltage on its clamps, in connection with physics of its structure. This property should be taken into account when analyzing the accumulated energy in SC and the accumulated charge in it.

In this experiment, SC and LC were charged from AB, while taking into account the change in the capacity of SC when the voltage at its terminals was changed. To this end, SC was represented by the amount of capacity $C_v(U) = k \cdot |U|$, which depends on the voltage on its terminals and unchanged capacity C_1 = const [13]. Being based on the scientific research [6, 10, 13], it is known that the coefficient $k = dC_v/dt$ has the dimension [F/V], and with sufficient accuracy it can be taken as a constant for this type of SC.

The capacity of SC is determined using the formula:

$$C(U) = C_1 + k \cdot |U|. \tag{1}$$

In order to study the energy characteristics of SC, in [2, 10, 14] a scheme for the replacement of SC shown in Fig. 1 was proposed.

This scheme with sufficient accuracy reflects the energy processes in SC being charged from a source of constant EMF for the time interval up to 30 minutes.

According to Exp. (1), the first branch is represented by two parallel capacitors – a capacitor with a constant capacitance and a capacitor with capacitance depending on the voltage [10, 14]. Due to the small value of the time constant, capacitors in this unit are charged

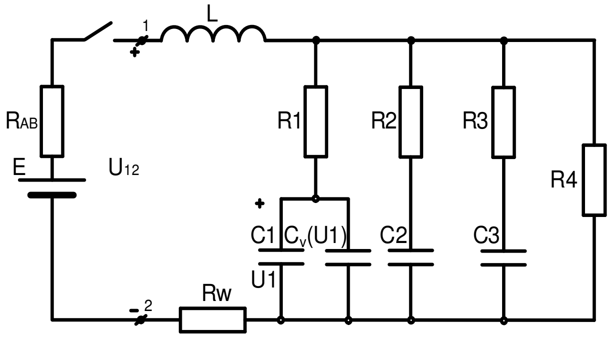

Fig. 1. Equivalent scheme of SC.

within the second range. The second branch with the constant elements C_2 and R_2 charges for a time interval equal to minutes. The third branch with the most constant time and with constant elements C_3 and R_3 operates in a time interval of 10 minutes. To take into account the self-discharge of SC in the equivalent circuit, the resistance R_4 is added. The lithium-ion AB with its internal resistance R_{AB} plays role of the non-ideal source of EMF. In this scheme, the resistance of wires R_W and the inductance of the circuit L are taken into account.

The capacity of LC does not depend on the voltage of their charge U, and the energy accumulated in them at zero initial voltage is defined by the expression [2, 15]:

$$W_{LC} = CU^2/2. \tag{2}$$

If the charge of SC started at zero initial conditions, then the expression for the energy W_{SC} accumulated in this SC at a certain final voltage U_f has the form [2, 14]:

$$W_{SC} = \int_{t_i}^{t_f} U(t) \cdot i(t) \cdot dt = \int_{Q_i}^{Q_f} U \cdot dQ = \int_{U_i}^{U_f} U \cdot (C_1 + 2kU) \cdot dU = \\ = C_1 U^2/2 + 2kU^3/3. \tag{3}$$

Under nonzero initial conditions for the voltage on the terminals of SC, from the expression (3) it is possible to determine the change of energy [2, 14] in SC at the voltages from U_i up to U_f:

$$\Delta W_{SC} = \frac{C_1(U_f^2 - U_i^2)}{2} + \frac{2k(U_f^3 - U_i^3)}{3} = \\ = (U_f - U_i) \cdot \left[\frac{C_1(U_f + U_i)}{2} + \frac{2k(U_f^2 + U_f U_i + U_i^2)}{3} \right]. \tag{4}$$

The change of electric energy $\Delta W_{LC} = W_f - W_i$ in LC for the case of voltage values from U_i to U_f, one can determine as

$$\Delta W_{LC} = \frac{C_{LC} U_f^2}{2} - \frac{C_{LC} U_i^2}{2} = \frac{C_{LC}(U_f^2 - U_i^2)}{2} = \\ = \frac{C_{LC}(U_f + U_i)(U_f - U_i)}{2}. \tag{5}$$

When SC is charged from zero initial conditions under voltage up to $U_f = U_n/n$ (where U_n is the nominal SC voltage and $n \geq 1$), the expression for the energy accumulated in this SC can be written as

$$\Delta W_{SC} = \frac{C_1 U_f^2}{2} + \frac{2kU_f^3}{3} = \frac{C_1 U_n^2}{2n^2} + \frac{2kU_n^3}{3n^3}. \tag{6}$$

When LC is charged under these conditions, according to (5), we obtain the formula for calculating the dose of energy that enters LC

$$\Delta W_{LC} = \frac{C_{LC} U_f^2}{2} = \frac{C_{LC} U_n^2}{2n^2} \tag{7}$$

and write the difference of the energies accumulated in SC (6) and LC (7):

$$\Delta W = \Delta W_{SC} - \Delta W_{LC} = \frac{C_1 U_n^2}{2n^2} + \frac{2kU_n^3}{3n^3} - \frac{C_{LC} U_n^2}{2n^2}. \qquad (8)$$

According to Exp. (8), provided that the capacitance of LC C_{LC} is equivalent to the capacity of SC and that their voltages $U_{SC} = 0$ ($C_{LC} = C_{SC}$) is equal, one can write

$$\Delta W = \Delta W_{SC} - \Delta W_{LC} = \frac{2kU_n^3}{3n^3}. \qquad (9)$$

From the formulae (6)–(9), it is determined that in SC at a charge from the voltage $U_i = 0$ to $U_f = U_n/n$, it will accumulate more energy $\Delta W = \frac{2kU_n^3}{3n^3}$ than in LC (on condition $C_{LC} = C_1$).

To study the influence of the terminal conditions on the voltage on the terminals of the capacitors $U_f = U_n/n$ on the difference in the energies ΔW that enter SC and LC, the graphic dependence $\Delta W(U_f = U_n/n)$ is plotted in Fig. 2.

The analysis of the dependence of the difference between the energy ΔW entering SC and LC on the voltage on terminals of the capacitors at the end of the charge process $U_f = U_n/n$ shows that when U_f changes within the range from $U_n/50 = 0.046$ V to $U_n = 2.3$ V, the value of the energy difference ΔW lies within the range from the minimum value $\Delta W(U_n/50) = 0.012$ J to the maximum one $\Delta W(U_n) = 1541$ J. If $U_f = U_n/n \to 0$ the energy difference ΔW also approaches to 0, according to the expressions (8) and (9). When charging SC and LC from zero initial conditions to the voltage $U_f = U_n/4.6 = 0.5$, the difference in the energies accumulated in SC and LC is $\Delta W = 15.85$ J, and with the increase of the final voltage by 2 times (at $U_f = U_n/2.3 = 1$), the difference between energies will be higher than 8.08 times and will reach 128.1 J. The difference in the energy doses accumulated in SC and LC,

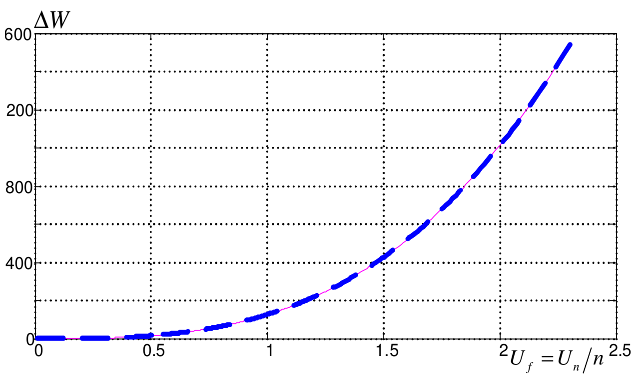

Fig. 2. The dependence of the influence of finite conditions under the voltage on terminals of the capacitors $U_f = U_n/n$ on the difference in the energies ΔW accumulated in SC and LC.

when charging under these conditions to the final voltage $U_f = U_n/1.15 = 2$, is ΔW = 1015 J and will be 64/04 times higher than the difference in the energy when charging to the final voltage $U_f = U_n/4.6 = 0.5$, while the value of the final voltage U_f is 4-fold increased.

From the expressions (6) and (7), it is possible to determine how many times the accumulated energy in SC will increase in comparison with that of LC, if charging from zero initial conditions to the voltage $U_f = U_n/n$, where the parameter $n \geq 1$:

$$\frac{\Delta W_{SC}}{\Delta W_{LC}} = \frac{\frac{C_1 U_n^2}{2n^2} + \frac{2kU_n^3}{3n^3}}{\frac{C_{LC} U_n^2}{2n^2}} = \frac{C_1}{C_{LC}} + \frac{4kU_n}{3nC_{LC}}. \qquad (10)$$

Assuming that $C_{LC} = C_1$, the expression (10) can be rewritten as:

$$\frac{\Delta W_{SC}}{\Delta W_{LC}} = 1 + \frac{4kU_n}{3nC_1}. \qquad (11)$$

According to Exp. (11), at charged from the voltage $U_i = 0$ to $U_f = U_n/n$, SC will accumulate in $\frac{\Delta W_{SC}}{\Delta W_{LC}} = 1 + \frac{4kU_n}{3nC_1}$ more energy than in LC (on condition $C_{LC} = C_1$). Under these conditions, a partial charge of SC and LC is a charge to the voltage $U_f = 2$ V. According to the formula (11), one can find the ratio of accumulated energies in the capacitors:

$$\frac{\Delta W_{SC}}{\Delta W_{LC}} = 1 + \frac{2kU_n}{3C_1}. \qquad (12)$$

After substituting the value of the SC nominal voltage, which is equal to the voltage AB $U_n = E = 2.3$ V, the coefficient $k = dC(U)/dU \approx \text{const}$ having the dimension [F/V] and the value of the voltage independent of the part of the SC capacitance $C_1 = 270$ in the expression (12), the ratio of the energies accumulated in capacitors will be 2.08.

To study the energy characteristics of circuits of the charge of SC from zero initial conditions to voltage $U_f = U_n/n$, it is necessary to consider the relationship between the ratios of energies accumulated in SC and LC (12) during their charging to the value of the final voltage on the terminals of capacitors $\frac{\Delta W_{SC}}{\Delta W_{LC}}\left(U_n/n\right)$. Fig. 3 shows the corresponding dependence.

Analysis of the dependence $\frac{\Delta W_{SC}}{\Delta W_{LC}}\left(U_n/n\right)$ in Fig. 3 confirms that, when charging of SC and LC from zero initial conditions to the voltage $U_f = U_n/n$, the ratio of energies accumulated in the capacitors in this case varies within the range from 1,04 for $U_f = U_n/50 = 0.046$ to 3.16 for $U_f = U_n = 2.3$. The analytical dependence (11)

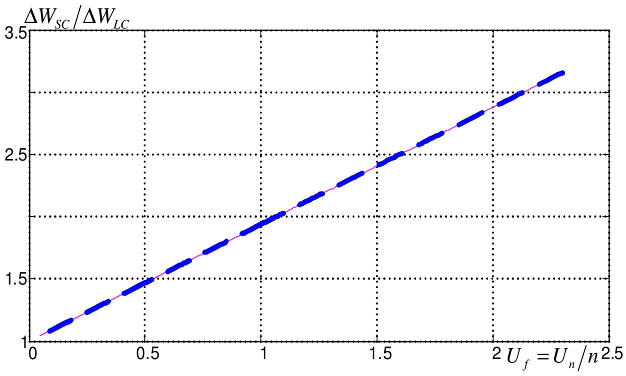

Fig. 3. The ratio of energies accumulated in SC and LC for the time of charge to the value of the final voltage on the terminals of capacitors $\frac{\Delta W_{SC}}{\Delta W_{LC}}\left(U_n/n\right)$.

confirms that the larger the value $U_f = U_n/n$, the higher is the ratio of energies accumulated during the charge under these conditions (for $U_f = 1$ this ratio is 1.94, and for $U_f = 2$ it is 2.88). These dependences (Fig. 1 to Fig. 3) should be taken into account when analyzing energy processes in the charge circuits of SC.

Consider the energy characteristics of charging SC and LC from a constant voltage source, when the initial conditions change (U_{0SC} = var and U_{0C} = var in the range up to $0.99U_n$).

Consider the dose of energy that enters LC, during its charge from voltage $U_f = U_n/n$ to $U_f = U_n = U_{AB}$. According to expression (5), one can obtain:

$$\Delta W_{LC} = \frac{C_{LC}\left(U_f^2 - U_i^2\right)}{2} = \frac{C_{LC}\left(U_n^2 - \left(\frac{U_n}{n}\right)^2\right)}{2} = \frac{C_{LC}\left(U_n + \frac{U_n}{n}\right)\left(U_n - \frac{U_n}{n}\right)}{2}. \quad (13)$$

If the charge of SC began at a nonzero initial voltage $U_f = U_n/n$, then, taking into account the expression (4), one can find the dose of energy ΔW_{SC} accumulated in SC during the charge to the voltage $U_f = U_n = U_{AB}$:

$$\Delta W_{SC} = \frac{C_1\left(U_f^2 - U_i^2\right)}{2} + \frac{2k\left(U_f^3 - U_i^3\right)}{3} = \frac{C_1\left(U_n^2 - \left(\frac{U_n}{n}\right)^2\right)}{2} + \frac{2k\left(U_n^3 - \left(\frac{U_n}{n}\right)^3\right)}{3}. \quad (14)$$

To simplify the expression and calculate the amount of accumulated energy in SC (14) during its charge, it is necessary to use the reduced multiplication formulae for the purpose of decomposition of the squared difference between these two expressions and the difference between the cubes of the two expressions, where do we have:

$$\Delta W_{SC} = \frac{C_1\left(U_n - \frac{U_n}{n}\right)\left(U_n + \frac{U_n}{n}\right)}{2} + \frac{2k\left(U_n - \frac{U_n}{n}\right)\left(U_n^2 + U_n\frac{U_n}{n} + \left(\frac{U_n}{n}\right)^2\right)}{3} = \left(U_n - \frac{U_n}{n}\right) \times \left(\frac{C_1\left(U_n + \frac{U_n}{n}\right)}{2} + \frac{2k\left(U_n^2 + U_n\frac{U_n}{n} + \left(\frac{U_n}{n}\right)^2\right)}{3}\right). \quad (15)$$

We will record the difference in the energies accumulated in LC (13) and SC (14), when charging from the initial voltage $U_f = U_n/n$ up to the voltage $U_f = U_n$:

$$\Delta W = \Delta W_{SC} - \Delta W_{LC} = \frac{C_1\left(U_n^2 - \left(\frac{U_n}{n}\right)^2\right)}{2} + \frac{2k\left(U_n^3 - \left(\frac{U_n}{n}\right)^3\right)}{3} - \frac{C_{LC}\left(U_n^2 - \left(\frac{U_n}{n}\right)^2\right)}{2}. \quad (16)$$

Assuming that the LC capacitance C_{LC} is equivalent to that of SC (provided its voltage is equal to $U_{SC} = 0$ ($C_{LC} = C_1$)), the expression (16) for determining the difference in the energy accumulated in SC and LC can be written as:

$$\Delta W = \Delta W_{SC} - \Delta W_{LC} = \frac{2k\left(U_n^3 - \left(\frac{U_n}{n}\right)^3\right)}{3} = \frac{2kU_n^3\left(1 - \frac{1}{n^3}\right)}{3}. \quad (17)$$

According to Exp. (17), the difference in energy doses ΔW accumulated in SC and LC, during the charge from the initial voltage $U_i = U_n/n$ to the voltage $U_f = U_n$, will be equal to $2kU_n^3\left(1 - \frac{1}{n^3}\right)/3$ for $C_{LC} = C_1$.

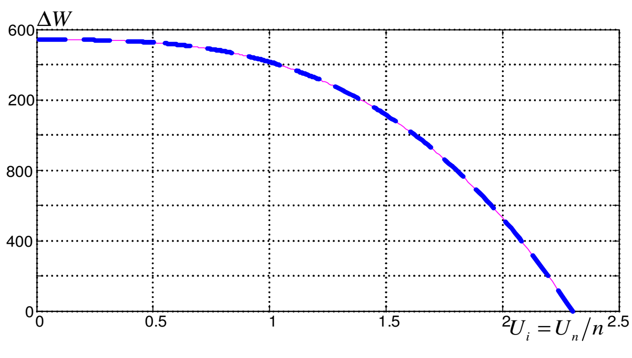

Fig. 4. Dependence of the difference between energies ΔW accumulated in SC and LC, in the process of charge from the initial voltage $U_f = U_n/n$ up to the final voltage $U_f = U_n$.

The dependence of difference in energy ΔW (17) accumulated in SC and LC, in the process of charge from the initial voltage $U_f = U_n/n$ up to the final voltage $U_f = U_n$, is depicted in Fig. 4.

Nonlinear dependence of the difference between energies ΔW accumulated in SC and LC for the time of charge under the given conditions of Fig. 4 was investigated in the range of change of the initial voltage from $U_i = 0$ to $U_i = U_n$. At the initial voltage $U_i \le 0.17$V, the difference in the energy accumulated in the capacitors will be maximum and equal to 1541 J. If the voltage on the terminals of SC and LC before the charge begins to be $U_i = 2$ V, then the difference in the energies accumulated in the capacitors under these charge conditions would be $\Delta W(U_i = 2) = 527.8$ J. In the case of charging SC and LC from the initial voltages on the terminals $U_i = 1.5$ V and $U_i = 1$ V, the difference between the energies accumulated in the capacitors will be 1114 and 1414 J, that is 2.11 and 2.68 times higher than at the charge from the voltage $U_i = 2$ V.

In order to determine how many times the dose of energy entering SC ΔW_{SC} is higher than the dose of energy accumulated in LC ΔW_{LC}, it is necessary to calculate their ratio (13)–(15) depending on the parameter n under these conditions of charging.

$$\frac{\Delta W_{SC}}{\Delta W_{LC}} = \frac{\left(U_n - \frac{U_n}{n}\right)\left[\frac{C_1\left(U_n + \frac{U_n}{n}\right)}{2} + \frac{2k\left(U_n^2 + U_n\frac{U_n}{n} + \left(\frac{U_n}{n}\right)^2\right)}{3}\right]}{\frac{C_{LC}\left(U_n - \frac{U_n}{n}\right)\left(U_n + \frac{U_n}{n}\right)}{2}} =$$

$$= \frac{\frac{C_1\left(U_n + \frac{U_n}{n}\right)}{2} + \frac{2k\left(U_n^2 + U_n\frac{U_n}{n} + \left(\frac{U_n}{n}\right)^2\right)}{3}}{\frac{C_{LC}\left(U_n + \frac{U_n}{n}\right)}{2}} =$$

$$= \frac{\frac{C_1\left(U_n + \frac{U_n}{n}\right)}{2}}{\frac{C_{LC}\left(U_n + \frac{U_n}{n}\right)}{2}} + \frac{\frac{2k\left(U_n^2 + U_n\frac{U_n}{n} + \left(\frac{U_n}{n}\right)^2\right)}{3}}{\frac{C_{LC}\left(U_n + \frac{U_n}{n}\right)}{2}} =$$

$$\frac{C_1\left(U_n + \frac{U_n}{n}\right)}{C_{LC}\left(U_n + \frac{U_n}{n}\right)} + \frac{4k\left(U_n^2 + U_n\frac{U_n}{n} + \left(\frac{U_n}{n}\right)^2\right)}{3C_{LC}\left(U_n + \frac{U_n}{n}\right)} = \qquad (18)$$

$$= \frac{C_1}{C_{LC}} + \frac{4kU_n^2\left(1 + \frac{1}{n} + \frac{1}{n^2}\right)}{3C_{LC}U_n\left(1 + \frac{1}{n}\right)} = \frac{C_1}{C_{LC}} + \frac{4kU_n\left(\frac{1}{n^2} + \frac{1}{n} + 1\right)}{3C_{LC}\left(\frac{1}{n} + 1\right)}.$$

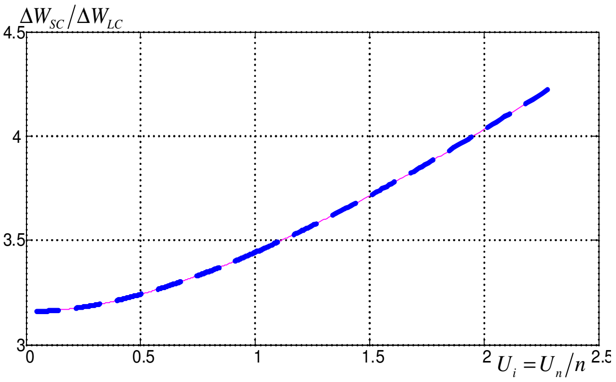

Fig. 5. Dependence of the ratio of energy doses accumulated in SC and LC during the charge from the voltage $U_i = U_n/n$ to $U_f = U_n$ on the value of initial voltage on the terminals of capacitors.

A separate case for Exp. (18) is the charge of SC and LC from the initial voltage $U_f = U_n/n$ to the voltage $U_f = U_n$, provided that the capacitance of LC C_{LC} is equivalent to the capacitance of SC (provided that its voltage is $U_{SC} = 0$ ($C_{LC} = C_1$) [2]). Under these conditions, the expression (18) should be rewritten in the form:

$$\frac{\Delta W_{SC}}{\Delta W_{LC}} = 1 + \frac{4kU_n\left(\frac{1}{n^2} + \frac{1}{n} + 1\right)}{3C_{LC}\left(\frac{1}{n} + 1\right)}. \qquad (19)$$

From the dependence in Fig. 5, it is evident that at the initial voltage $U_i = U_n/n \to 0$ the ratio of energies accumulated in SC and LC during the charge from the voltage $U_i = U_n/n$ to $U_f = U_n$ will be minimal and equal to 3.16. For the initial voltage $U_i = U_n/n = 1\,\text{V}$, the ratio of energies accumulated under these conditions is 3.44, and at voltage $U_i = U_n/n = 2\,\text{V}$ the ratio of energies will increase by 17.15%. The maximum value of the energy ratio can be obtained at the charge of SC and LC from the initial voltage $U_i = U_n/_{n=1.01} = 2.28$ V to $U_f = U_n$. Accordingly, with $U_i = 2.28$ V, this ratio is 4.22. A significant advantage of SC is that its specific energy in several hundred times is greater than the specific energy of LC [2, 14].

The dose of energy W_{AB} taken from AB with the voltage U_{AB} can be found using the formula:

$$W_{AB} = \int_{t_i}^{t_f} U_{AB} \cdot i(t) \cdot dt\,. \qquad (20)$$

The energy transfer coefficient η [2, 13] is equal to the ratio of the energy received in LC or in SC to the energy taken from AB for the entire charge time:

$$\eta_{LC} = \left(W_{LC}(t_i) - W_{LC}(t_f)\right)/W_{AB}\,, \qquad (21)$$

$$\eta_{SC} = \left(W_{SC}(t_i) - W_{SC}(t_f)\right)/W_{AB}\,, \qquad (22)$$

where $W_{LC}(t_i)$, $W_{SC}(t_i)$, $W_{LC}(t_f)$, $W_{SC}(t_f)$ is the energy accumulated in LC and SC in accordance with the switching and after the end of the transition process of the charge from AB; $W_{AB} = W_{AB}(t_i) - W_{AB}(t_f)$ – energy given by AB during the transition process.

From the expressions (2) to (7) and (20), one can obtain the energy of losses [2, 13] in the circuit of charge of LC and SC from AB. The energy of losses is the difference between the energy given by AB and the energy received in the process of charge by LC or SC:

$$W_{lossesLC} = \left(W_{AB}(t_i) - W_{AB}(t_f)\right) - \left(W_{LC}(t_f) - W_{LC}(t_i)\right) = = W_{AB}(1 - \eta_{LC}), \quad (23)$$

$$W_{lossesSC} = \left(W_{AB}(t_i) - W_{AB}(t_f)\right) - \left(W_{SC}(t_f) - W_{SC}(t_i)\right) = = W_{AB}(1 - \eta_{SC}). \quad (24)$$

Thus, in analyzing the energy processes of linear capacitors charged to any voltage, one can limit himself by a constant value of their capacitance C, and in the analysis of these processes in SC [2, 13, 14] charged to some voltage U_1, it is necessary to calculate their capacitances as a function $C(U_1)$ for the voltage value U_1.

According to the expressions (2) to (7), SC accumulates more energy than LC charged to the same voltage, therefore, when discharging SC and LC, having the same internal resistances ($R_{LC} = R_{SC}$) and being charged to the same voltage, SC will give the load resistance R_n more energy and with greater pulse power than LC [2].

From the formulae (2) to (7), we can obtain the expression for the LC capacitance C_W, which is equivalent to the capacitance of SC [10], provided that the energies accumulated in them ($W_{LC} = W_{SC}$) and the voltages on the plates ($U_{LC} = U_{SC}$) are equal:

$$C_W = C_1 + 4kU/3. \quad (25)$$

Note that when the voltage and charge equations of LC and SC are equal $U_{LC} = U_{SC}$ and $Q_{LC} = Q_{SC}$, their capacity also should be equal to the determination of the capacitance of any capacitor [2, 14], therefore the capacitance of LC C_Q is equivalent to the total capacitance of SC, provided that the charges of LC and SC in accord with the expression (1) will be equal to

$$C_Q = C(U) = C_1 + kU. \quad (26)$$

In the discharge of SC and equivalent to the initial energy of LC from the initial voltage U_0 to the half-voltage $U_0/2$, the energy remaining in each of the capacitors after their discharge, taking into account (3) and (25), can be calculated as follows [2]:

$$W_{SC}(t_f) = \frac{C_0 (U_0/2)^2}{2} + \frac{kU_0^3}{12}, \quad (27)$$

$$W_{LC}(t_f) = \frac{C_W (U_0/2)^2}{2} = \frac{C_0 (U_0/2)^2}{2} + \frac{kU_0^3}{6}. \quad (28)$$

Comparing (27) and (28), we can conclude that after discharge from U_0 to $U_0/2$, in SC, there is less (at a load given more) energy than in LC by the factor $kU_0^3/12$. For example, SC which according to [2, 10, 14] has the parameters C_0 = 270 F, k = 190 F/V, U_0 = 2,3 V at full charge has the capacitance C_{SC} = 707 F. Then the energy equivalent to it is the capacitance of LC C_W = 852.7 F. The initial energies of SC and LC will be equal to 2255.3 J, and after discharge from U_0 to $U_0/2$ the capacitors will have $W_{SC}(t_f)$ = 371.2 J and $W_{LC}(t_f)$ = = 563.8 J, respectively, 16.5 and 25% of the initial energy.

The advantage of SC is also that its specific energy is several hundred times higher than that of LC [2]. That is, when you should accumulate the same energies, the weight of the SC battery will be several hundred times less than that of LC. By specific energy, industrial designs of the SC approach to modern lead AB [5], and their specific power more than 20 times exceeds the specific capacity of serial lithium-ion batteries [2, 5].

3. Conclusions

1. In this work, the analysis of energy processes in combined power supplies from SC and LC, with their charge from a non-ideal source of constant voltage, has been performed, for example, with a lithium-ion battery of accumulators. The peculiarities of influence on the accumulated energy of the initial and final voltages on the terminals of capacitors and their capacities, taking into account the dependence of the capacity of SC on the voltage value, have been ascertained.

2. To study the energy characteristics of circuits of the charge of SC from zero initial conditions to voltage $U_f = U_n/n$, the dependence of the ratio of energies accumulated in SC and LC during the charge on the value of the final voltage $U_f = U_n/n$ on the terminals of the capacitors is considered. Analysis of this dependence confirms that the ratio of energies accumulated in this case in the capacitors varies within the range of values from 1.04 for U_f = 0.046 to 3.16 for U_f = 2.3.

3. The analytical dependence has confirmed that the larger the value $U_f = U_n/n$, the greater the ratio of energies accumulated during the charge under these conditions.

4. The ratio of energy doses accumulated in SC and LC, in the process of charge from the voltage $U_f = U_n/n$ up to the voltage $U_f = U_n$, depending on the value of the initial voltage on the terminals of the capacitors $U_f = U_n/n$, has been analyzed. At the initial voltage $U_i \to 0$, this ratio of energies is minimal and equal to 3.16. The maximum value of the energy ratio can be obtained at the charge of these SC and LC from the initial voltage U_i = 2.28 V to $U_f = U_n$. Accordingly, with U_i = 2.28 V, the ratio is 4.22.

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