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Characteristic frequencies of transverse electric modes in a double negative slab waveguide with Kerr-type nonlinearity

A. Yalçınkaya[1], A. Çetin[2]

[1]*Graduate School of Natural and Applied Sciences, Eskişehir Osmangazi University, Meşelik Kampüsü, Odunpazarı 26040, Eskişehir, Türkiye*
E-mail: dr.ahmetyalcin@gmail.com
[2]*Faculty of Science, Eskişehir Osmangazi University, Eskişehir, Türkiye, Meşelik Kampüsü, Odunpazarı 26040, Eskişehir, Türkiye*
E-mail: acetin@ogu.edu.tr

Abstract. The evolution equation for guided transverse electrical modes in a slab waveguide having a double negative core is derived for the electrical field. Special solutions of the dispersion equation for the case with double positive symmetric cladding are searched for. Relevant eigenmodes are formulated in terms of characteristic frequencies as a new method, and these frequencies corresponding to the oscillating guided modes are found for different wave numbers and core widths assuming the lossless Drude model can be used for the core medium with a Kerr-type nonlinearity. Results show that normalized characteristic frequencies increase with increasing wave numbers and mode numbers.

Keywords: slab waveguide, transverse modes, characteristic frequencies, Kerr-type nonlinearity, metamaterials.

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1. Introduction

Negative index materials (NIMs), also known as left-handed materials (LHMs) or metamaterials (MTMs) were first introduced by Veselago [1], and experimentally verified by Shelby *et al.* [2]. Pendry *et al.* showed that NIMs can be designed artificially [3], and carried out exclusive research on NIMs and negative refraction [4–6]. NIMs are different from conventional right-handed material, and have negative permittivity and permeability [7, 8] simultaneously so they are referred to as double-negative medium. They are artificial materials that are not available in nature and have numerous features. It is known from the literature that various properties and applications of LHM, or MTMs, have been researched for more than several decades [9–19]. In NIMs, negative refraction is theoretically proposed as a result of relevant calculations and is also experimentally observed [20–23].

Waveguides as essential elements of photonics have a large background in research regarding electromagnetic waves and modes. Various analytical and numerical studies of waveguide theory, waveguides, and numerous applications are available in the literature [24–27]. In cases where metamaterial or LHM are used as a medium for waveguides with double positive (DPS), single negative (SNG), or double negative (DNG) variations for the claddings and substrates as different options, they show interesting properties [28–30].

The Kerr media have third-order nonlinearity due to the quadratic effect of an electric field as a nonlinear factor in the evolution equation of the propagation which is known as the Kerr effect. It is one of the widely investigated media in nonlinear optics and integrated optics [31–35].

Planar waveguides with Kerr-type nonlinear media make possible the existence of modes, which are not present in natural material [36–38]. Various slab waveguide structures with DNG core and DPS, SNG, and DNG claddings or substrates were subject to studies and research before, [40–46], and some of the investigations done include multilayer waveguides [47–50].

In this paper, we present the evolution equation for a DNG slab waveguide with Kerr-type nonlinearity, mention shortly the general solutions, and present some special solutions considering DPS cladding and substrate, searching for the transverse electric (TE) modes, and compare the results based on analytic and numerical results. Finally, we look for the characteristic frequencies corresponding to the eigenmodes as solutions of the dispersion relation. Characteristic frequencies directly obtained from the dispersion equation are not tried before to the best of our knowledge, and this is a new application.

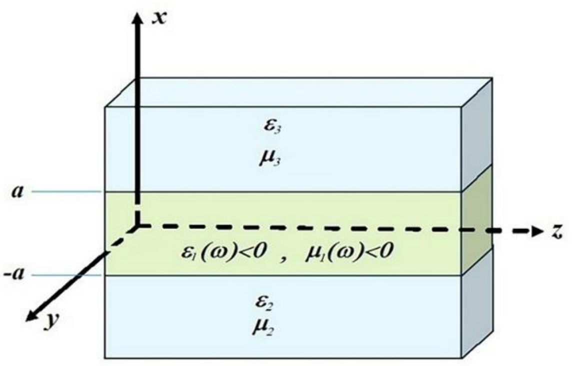


Fig. 1. Slab waveguide with negative-index core and conventional claddings.

2. Evolution equation for a slab waveguide with a Kerr-type core

As known from photonics, in a slab waveguide with wave propagation in *z* direction, and *x* as the transverse direction assuming the refractive index to change only in that direction, TE modes will have just a *y*-component depending spatially only in *x*.

Fig. 1 shows such a slab waveguide structure with a negative-index core with a width of 2*a* and *z* axis in the center of that core. It is worth not forgetting that the waveguide is assumed to be infinite in *y* and *z* directions and that we restrict ourselves to the linear conventional materials for cladding and substrate.

We use for the core material a medium that shows Kerr-type nonlinearity so that the nonlinear term $\mathbf{D}_{NL} = \alpha\mathbf{E}|\mathbf{E}|^2$ must be considered in expressions and added as the nonlinear term of the displacement field [35]. In this case, the refractive index of the core being nonlinear will be expressed as $n_1^2 = \varepsilon_1\mu_1 = \alpha_1|E_{y1}|^2$, where E_{y1} is the *y*-component of the field in the core, ε_1, and μ_1 are the negative permittivity and permeability, and α_1 is the nonlinear coefficient [46, 50].

In this waveguide with 2*a* width, *i.e.* $-a \le x \le a$, the component of the electric field in the *y* direction can be written as $\mathbf{E} = \hat{\mathbf{y}}E_y(x)\left[e^{j\beta z} + e^{-j\beta z}\right]e^{-j\omega t}$, where $e^{-j\omega t}$ is the time-harmonic excitation [40]. Here, β is the propagation constant in the longitudinal direction and ω is the angular frequency. Without loss of generality, eliminating the complex conjugate since the result will not change, the equation will be $\mathbf{E} = \hat{\mathbf{y}}E_y(x)\,e^{j(\beta z - \omega t)}$.

Considering the Kerr effect, introducing the dimensionless variables $u = k_0 x$, $q = \beta/k_0$ and remembering that $k_0 = \omega/c$ is the wave number in vacuum with *c* being the speed of light in vacuum, the evolution equation for $E_y(x)$ is derived for the TE mode as [50]

$$\frac{\partial^2 E_{yi}}{\partial u^2} + \left[\kappa_i^2 + \eta_i |E_{yi}|^2\right]E_{yi} = 0\,, \qquad (1)$$

with *i* = 1, 2, 3 for core, substrate, and cladding, respectively. Eq. (1) is a nonlinear Helmholtz equation in one dimension [31, 35]. Here, η_1 is a coefficient representing the Kerr-effect contribution to the core depending on the permeability μ_1. For the general case with Kerr medium in all three layers, corresponding coefficients for cladding and substrate will be η_3 and η_2, respectively. The coefficients η_i are determined observing the effects of the nonlinear medium and are related to frequency-dependent permittivity and permeability.

The equation for the TM mode can be obtained similarly by making the necessary transformation in interchanging permittivity ε_1 and permeability μ_1, respectively, with μ_1 and ε_1, and E_{y1} with H_{y1} [36].

If all three layers are nonlinear and dispersive, the transverse wave parameter $\kappa_1 = \sqrt{\varepsilon_1(\omega)\mu_1(\omega) - q^2}$ as well as the parameters $\kappa_3 = \sqrt{q^2 - \varepsilon_3(\omega)\mu_3(\omega)}$ and $\kappa_2 = \sqrt{q^2 - \varepsilon_2(\omega)\mu_2(\omega)}$ are dependent on the wave number, with permittivity and permeability being dependent on the frequency.

3. General solutions of the evolution equation

The solutions for oscillating guided modes are possible if $n_1^2 > n_3^2$ and $n_1^2 > n_2^2$, where $n_2 > n_3$. For the slab waveguide described in Fig. 1, the cladding and the substrate are conventionally linear materials. Hence, the coefficients η_3 and η_2 become both 0, and $\eta_1 = \alpha_1$ satisfying the equation $\varepsilon_1 = \varepsilon_{1lin} = \alpha_1|E_{yi}|^2$, where ε_{1lin} is the linear part of the permittivity. The electric field profiles for all three regions can be written as [42, 44]

$$E_{y3}(u) = E_{03}e^{-\kappa_3(u-a)}, \qquad (2a)$$

$$E_{y1}(u) = E_{01}\cos(\kappa_1 u + \phi), \qquad (2b)$$

$$E_{y2}(u) = E_{02}e^{-\kappa_2(u+a)}, \qquad (2c)$$

where E_{03}, E_{01}, and E_{02} are field amplitudes in the cladding, core, and substrate, respectively.

3.1. Solutions for guided modes

For oscillating guided modes, κ_1 needs to be real so that $\varepsilon_1\mu_1 = n_1^2 > q^2$. Phase shift φ in the core can be expressed as $\phi = a\kappa_1 + \arctan(-\mu_1\kappa_2/\kappa_1\mu_2)$.

At the interfaces, it is obvious that electric fields must be identical, and that their first derivatives should be continuous. Consequently, at the interface between core and cladding, in other words, when *u* = *a*, boundary conditions $E_{y3} = E_{y1}$ and $\mu_1[dE_{y3}/du] = \mu_3[dE_{y1}/du]$ must be satisfied [51].

Similarly, at the interface between core and substrate, $E_{y2} = E_{y1}$, and their first derivatives should be continuous. Applying boundary conditions leads to the dispersion equation given as

$$2\kappa_1 a = m\pi + \tan^{-1}\left(\frac{\mu_1\kappa_3}{\mu_3\kappa_1}\right) + \tan^{-1}\left(\frac{\mu_1\kappa_2}{\mu_2\kappa_1}\right). \qquad (3)$$

This equation is transcendental and cannot be solved analytically. Newton–Raphson or Bisection methods can be used to solve this problem for κ_1, determining the eigenmodes.

Eqs. (2) determine the complete field profile for all three layers. For a numerical result or a graphical demonstration, the field amplitudes need to be found for relevant "*a*" and "–*a*" values. A normalization using these amplitudes is necessary for obviate of very small numbers.

The electric field profile for the above solution with normalized fields $E_{y3n}(u) = b_3(E_{y3}/E_{03})$, $E_{y1n}(u) = b_1(E_{y1}/E_{01})$, and $E_{y2n}(u) = b_2(E_{y2}/E_{02})$ is given for $a = 1$ in Fig. 2 where b_i are normalization constants depending on the amplitudes.

From the boundary conditions, using continuity, we see that $E_{03} = E_{01}(-\kappa_1)\sin(a\kappa_1 + \phi)$ and similarly $E_{02} = E_{01}(-\kappa_1)\sin(-a\kappa_1 + \phi)$, which results in $\text{sign}(b_3) = -\text{sign}(b_2)$. For the field profile of TE_1 in the symmetric case, we take as an example the parameters $\kappa_3 = \kappa_2 = 1$, and a wave number corresponding to $\varepsilon_2 = \varepsilon_3 = \mu_2 = \mu_3 = -\mu_1 = 1$, which is common in the literature [35, 36, 42]. The transverse wave parameter is taken as $\kappa_1 = 1.5$, and the boundary value at maximum shift, *i.e.*, $\phi = -\pi/2$ is used for easiness without losing generality as $\phi = -m\pi/2$ is valid at the boundaries for other modes as well. In this example, with *m* as the mode number, $b_2 = -b_3 = -b_1 = (-1)^m$ is taken just for simplicity considering even modes and odd modes to show different behaviors. From Eqs. (2) and (3), we see that with the cladding being symmetric and double positive for the above parameters, only TE_1 exists, which is a special case determined by the selected parameters for permittivity, permeability, and transverse wave number. If we take $\kappa_1 = 3$, other parameters being the same, this time TE_2 will exist. Other profiles, which can be demonstrated for different combinations of permittivity, permeability, and transverse wave numbers result in more than one TE profile.

Our investigation is to find the characteristic frequencies allowing the oscillating guided modes in the waveguide. For this solution, we need to use a model regarding the refractive index dependence on the angular frequency, and, consequently, on the wave number.

3.2. Solutions for frequencies using models of dispersive cases

In media with losses, both the permittivity and the permeability are complex. In other words, they can be written with their real and imaginary parts as $\varepsilon_1(\omega) = \varepsilon_{1\text{Re}}(\omega) + j\varepsilon_{1\text{Im}}(\omega)$ and $\mu_1(\omega) = \mu_{1\text{Re}}(\omega) + j\mu_{1\text{Im}}(\omega)$ [43] and the main parameter responsible for the losses is generally the imaginary part of the permeability.

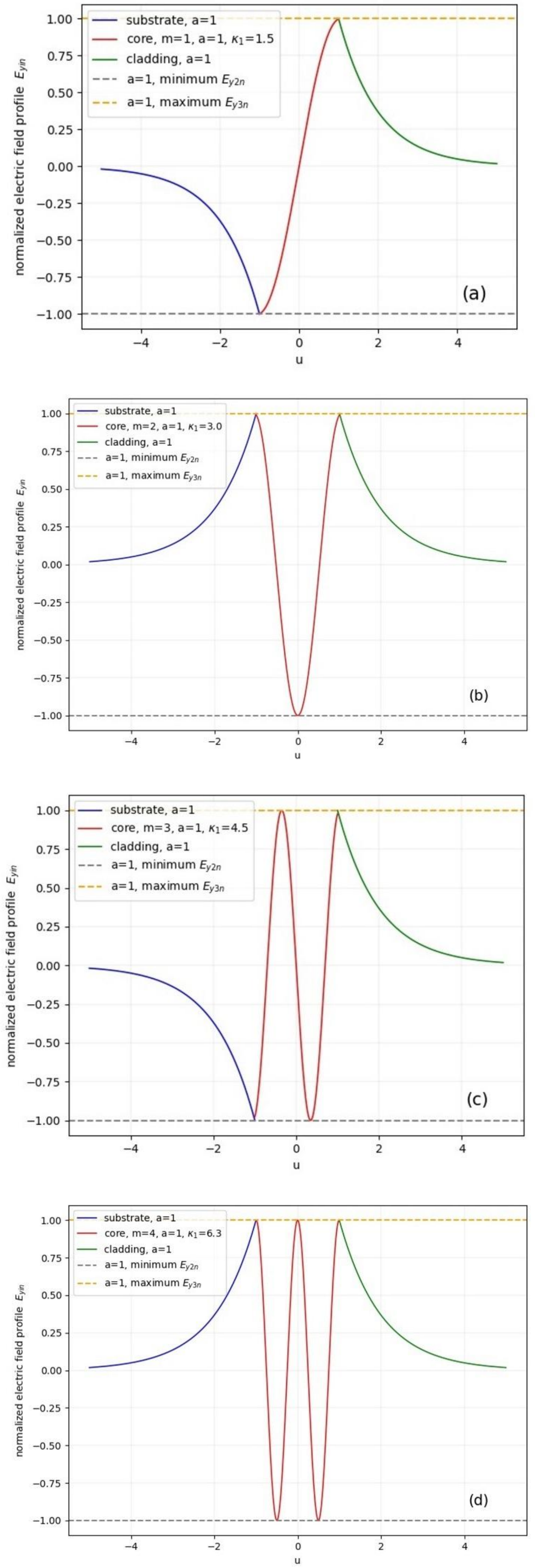


Fig. 2. Normalized electric field profiles for $a = 1$; a) for TE_1 with $\kappa_1 = 1.5$, b) for TE_2 with $\kappa_1 = 3.0$, c) for TE_3 with $\kappa_1 = 4.5$, and d) for TE_4 with $\kappa_1 = 6.3$.

In cases of Kerr-type nonlinearities, the Drude model can be used for which permittivity and permeability are expressed respectively in a general way for the lossy media as $\varepsilon_1(\omega)=1-\omega_p^2/\omega(\omega+j\gamma_p)$, $\mu_1(\omega)=1-F\omega_p^2/\omega(\omega+j\gamma_p)$ in the frequency domain [16, 41], where γ_p is the damping coefficient responsible for the losses. Its value differs according to the material in question. The so-called filling parameter F, a constant indicating the volumetric ratio of the magnetic portion gives an idea about the ratio between permittivity and permeability and is usually taken as 0.56 in the literature [27]. It can be defined as the volume occupied by split rings to the unit cell volume for metamaterials.

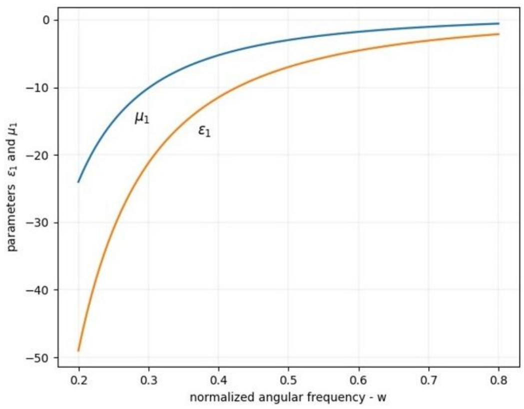
For the solution in this paper, the high-frequency region is considered so that $\omega >> \gamma_p$, and the lossless Drude model is assumed, *i.e.*, the damping ratio is taken as zero ($\gamma_p = 0$) so that $\varepsilon_1(\omega)=1-\omega_{ep}^2/\omega^2$, $\mu_1(\omega)=1-\omega_{mp}^2/\omega^2$ can be used for permittivity and permeability, respectively. The losses are actually important in nonlinear media, but especially in microwave region the losses can be negligible. Nowadays, it is possible to artificially manufacture metamaterials, and especially for very low values of the imaginary parts of ε and μ, losses can be considered negligible so that the lossless Drude model assumption is acceptable [52]. Using this approach, the solutions for the characteristic wave numbers or corresponding frequencies are searched for regarding the assumption that electronic plasma frequency is related to the magnetic plasma frequency as $\omega_p^2=\omega_{ep}^2=2\omega_{mp}^2$ considering the usual parameter F to be approximately the same as in the lossless Drude model complying with the value given in the literature. As permittivity and permeability are both negative for the metamaterials in question, it is obvious that $\omega_{mp} > \gamma_p$.

Defining a normalized frequency $w = \omega/\omega_{mp}$, we see first how the relation between the constants and w are for the lossless Drude model case with the plasma frequency assumptions above, and find the expression for κ_1. Fig. 3 shows the relation between normalized frequency and the parameters permeability and permittivity as well as its relation to longitudinal normalized wave number q and the transverse wave parameter κ_1. In our example, we assume the normalized wave number q can be taken independently of w for simplicity regarding that it is normalized with the vacuum wave number, and that special discrete values of q can be found.

To proceed for the solution, with $\kappa_1=\sqrt{(1-1/w^2)(1-2/w^2)-q^2}$, Eq. (3) is rewritten as

$$\frac{a}{|w|}\sqrt{(w^2-1)(w^2-1)-w^2q^2} =$$

$$= m\frac{\pi}{2}+\tan^{-1}\left(\frac{w^2-1}{|w|\sqrt{(w^2-1)(w^2-1)-w^2q^2}}\right), \qquad (4)$$



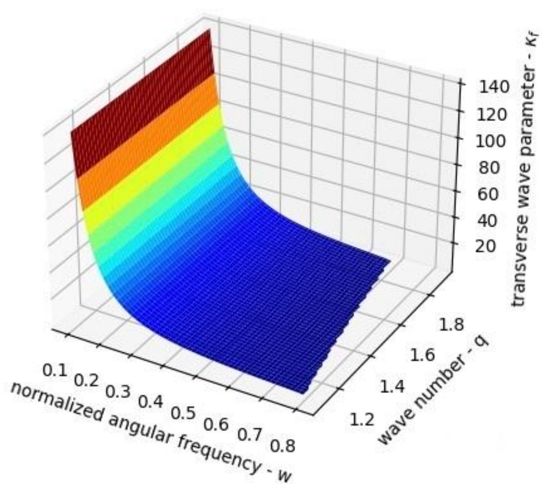


Fig. 3. Variables depending on normalized angular frequency w: a) permittivity ε_1 and permeability μ_1, b) 3-D graphics of dependences between q, w, and κ_1 for the theoretical assumption of continuous q values.

using the lossless Drude model for the core, and a symmetric cladding with $\kappa_3 = \kappa_2 = 1$ for simplicity. We look for solutions noting that it is possible for $\kappa_1 > 0$, if we are searching guided waves. This is valid for a certain interval of q values only. Otherwise, *i.e.*, if κ_1 is imaginary, the outcome will be the propagation of surface waves which are evanescent waves traveling along boundaries.

4. Numerical results

The graphical and numerical solution of Eq. (4) shows that frequencies for making possible the fundamental mode exist are not available, neither for different values of a nor for any value of the normalized wave number q as $\kappa_1 > 0$. The graphical solutions for $a = 1$ and $a = 2$ are given for TE_1 to TE_4 in Fig. 4.

The findings regarding the solution of Eq. (4) have shown the discrete nature of the characteristic frequencies for the eigenmodes to exist. While searching for solutions for $a = 1$ and $a = 2$ with the given discrete

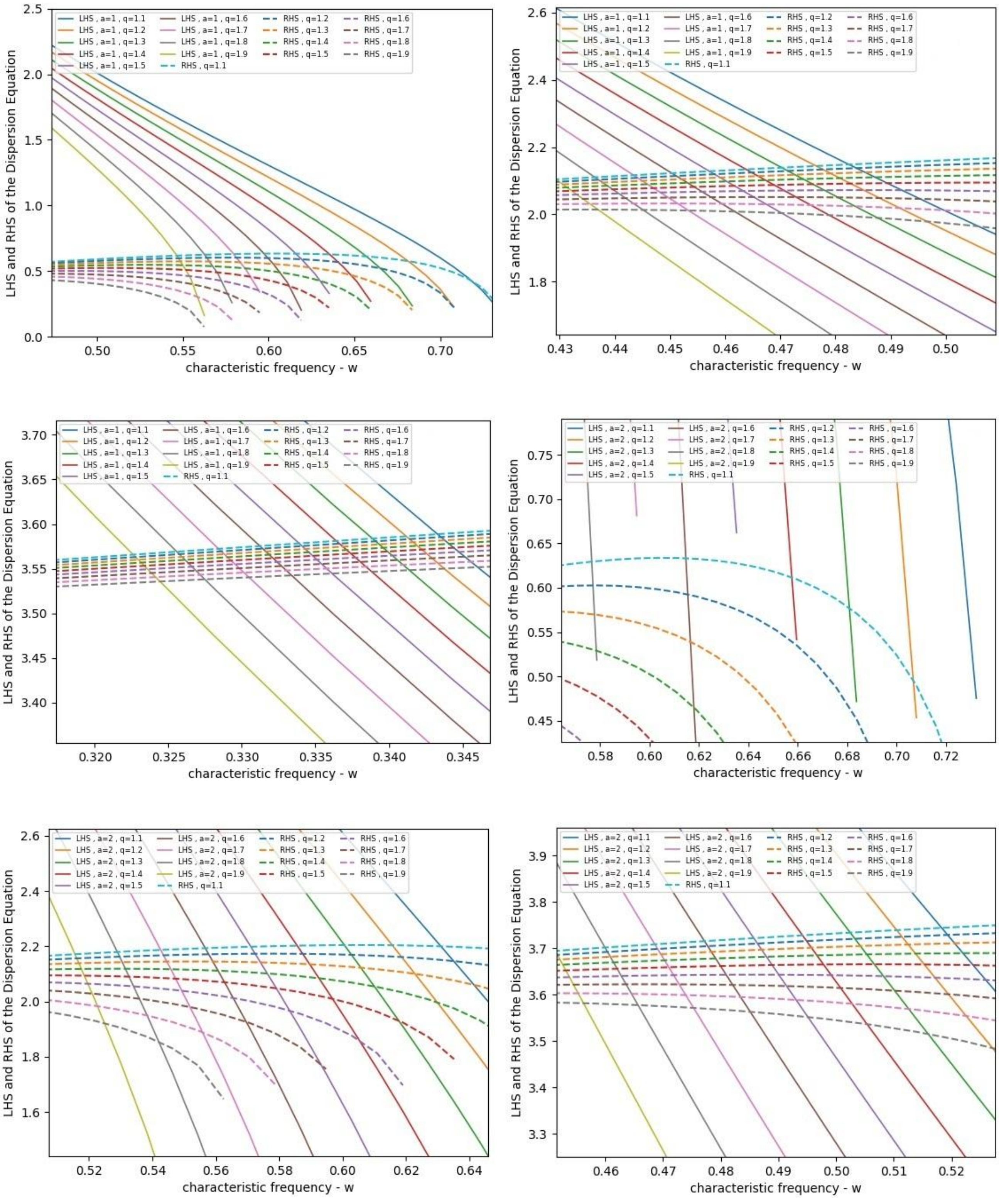


Fig. 4. The graphical solutions for TE mode frequencies of the dispersion equation for different wavenumbers and core widths. For a = 1: a) TE_1, b) TE_2, c) TE_3, for a = 2: d) TE_1, e) TE_2, f) TE_3. (Color online)

normalized wave numbers q = 1.1 to 1.9, we have seen that there is no converging result for the fundamental mode, and only two results for TE_1 in the case of the sensitivity equal to 10^{-6} are used in the calculations. A list of the characteristic frequencies considering $\omega_{mp} > \omega$ is given in Tables 1 and 2 below.

Table 1. Normalized characteristic frequencies of the eigenmodes for $a = 1$ with given q and m values. The abbreviation NA means "Not Available".

q	$m = 1$	$m = 2$	$m = 3$	$m = 4$
1.1	0.730182	0.477866	0.318248	0.231202
1.2	0.712638	0.475101	0.317393	0.230863
1.3	NA	0.472070	0.316463	0.230494
1.4	NA	0.468773	0.315459	0.230097
1.5	NA	0.465204	0.314381	0.229670
1.6	NA	0.461360	0.313229	0.229215
1.7	NA	0.457239	0.312003	0.228730
1.8	NA	0.452843	0.310704	0.228217
1.9	NA	0.448170	0.309332	0.227676

Table 2. Normalized characteristic frequencies of the eigenmodes for $a = 2$ with given q and m values. The abbreviation NA means "Not Available".

q	$m = 1$	$m = 2$	$m = 3$	$m = 4$
1.1	NA	0.662821	0.521938	0.411392
1.2	NA	0.650989	0.517378	0.409306
1.3	NA	0.638243	0.512435	0.407045
1.4	NA	0.624692	0.507117	0.404610
1.5	NA	0.610478	0.501435	0.402003
1.6	NA	0.595772	0.495402	0.399229
1.7	NA	0.580761	0.489036	0.396289
1.8	NA	0.565627	0.482360	0.393188
1.9	NA	0.550536	0.475397	0.389930

5. Conclusions

In this paper, a slab waveguide with DPS symmetrical cladding and DNG core has been taken as the guiding structure considering the Kerr-type nonlinearity effect. The guided TE modes in the core are investigated. After summarizing the evolution equation for the general Kerr-type NIM core and conventional cladding using the normalized variables, and touching briefly on its solutions, the dispersion equation for the symmetric cladding is given. Using the lossless Drude model for the permittivity and permeability, the solutions for the characteristic frequencies of the eigenmodes are searched for considering only oscillating modes avoiding solutions for surface waves. In the analysis, a new application is used and characteristic frequencies are found directly from the numerical solution of the dispersion equation. It is seen from the graphical solution that characteristic frequencies of the modes decrease with the increase in the wave and mode numbers. The first result shows that the fundamental mode vanishes as characteristic frequencies for that mode do not exist. Contrary to the wavenumbers, the increase in the core width causes an increase in the characteristic frequencies for the same mode and wave numbers.

The research for waveguides and especially metamaterials will for sure increase in numbers and the novel outcomes in both fields will not be surprising. There is, however, a trend of the waveguides investigation morely using conventional materials, and a trend of the study on metamaterials considering their structure, properties, and production methods. If one considers the various waveguides types with the metamaterial as a DNG medium for different core, cladding, and substrate structures, the further properties of electromagnetic waveguiding and application possibilities will be discovered surely. This is a field open both for improvement and development.

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Authors and CV

Ahmet Yalçınkaya, PhD Candidate in Physics at Eskişehir Osmangazi University, got his BS in Mechanical Engineering from Boğaziçi University in 1987, and MS in Robotics Engineering from Istanbul Technical University in 1991. Mainly worked as engineer, manager, or consultant in the industry for more than 30 years. Served as part-time project assistant, part-time lecturer, and volunteer technology consultant at some universities in Türkiye and abroad. Author of many publications in management, economics, and robotics. His research areas in Physics are nonlinear optics and metamaterials.
http://orcid.org/0000-0003-1537-3638

Ali Çetin, PhD, Assistant Professor at Eskişehir Osmangazi University, got his BS and MS in Physics from Anadolu University in 1988 and 1991, respectively, and PhD in Physics from Eskişehir Osmangazi University in 1998. He worked as a research assistant in the Physics Department of Eskişehir Osmangazi University from 1990 to 1998, where he joined the same department as a faculty member in 1998. Author of many publications. Advised various graduate thesis and dissertations. His research interests include photonics, nonlinear optics, and waveguide technology.
http://orcid.org/0000-0003-0468-8087

Authors' contributions

All the authors contributed equally to the manuscript.

Характеристичні частоти поперечних електричних мод у подвійному негативному планарному хвилеводі з керрівською нелінійністю

A. Yalçınkaya, A. Çetin

Анотація. Для електричного поля отримано рівняння еволюції напрямлених поперечних електричних мод у планарному хвилеводі з подвійним негативним сердечником. Здійснено пошук спеціальних розв'язків дисперсійного рівняння для випадку з подвійною позитивною симетричною оболонкою. Відповідні власні моди сформульовані в термінах характерних частот як новий метод, і ці частоти, що відповідають осцилюючим спрямованим модам, знайдені для різних хвильових чисел і ширин ядра, припускаючи, що модель Друде без втрат може бути використана для основного середовища з нелінійністю типу Керра. Результати показують, що нормовані характеристичні частоти зростають зі збільшенням хвильових чисел і номерів мод.

Ключові слова: планарний хвилевід, поперечні моди, характеристичні частоти, керрівська нелінійність, метаматеріали.