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Optics

Optical soliton perturbation with the concatenation model having multiplicative white noise by *F*-expansion

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Abstract. The paper investigates the concatenation model incorporating the Kerr law of self-phase modulation and Hamiltonian perturbations, along with the effect of multiplicative white noise. The *F*-expansion algorithm is implemented to derive the solutions for the model, thus yielding a full spectrum of optical solitons. The parameter constraints that naturally emerge from the scheme are also listed.

Keywords: solitons, white noise, *F*-expansion.

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1. Introduction

First introduced in 2014, the concatenation model has been the focus of ongoing research, initially investigating its integrability and exploring rogue wave phenomena [1, 2]. Since then, subsequent studies have delved deeper into various aspects of the model, including conservation laws, quiescent solitons with nonlinear chromatic dispersion (CD), trial equation methods, undetermined coefficients, and the application of Kudryashov's integration schemes to address issues related to differential group delay. These studies represent just a fraction of the broader research aimed at fully understanding the concatenation model [1–5].

This paper specifically examines the concatenation model under the influence of multiplicative white noise. It employs an analytical approach to explore the model and uncover new insights: the *F*-expansion method. This methodology facilitates the discovery of soliton solutions in the presence of white noise. Some preliminary approaches to the model were implemented earlier, which yielded soliton solutions [6–9]. By selecting specific parameter values, the paper reveals a wide spectrum of optical solitons as well as complexiton solutions. Notably, the study reveals that white noise primarily affects the phase component of the solitons. The detailed analysis and results, along with their derivations, are exhibited in subsequent sections.

1.1. Governing model

The model in [1] illustrates the stochastic perturbed concatenation in a dimensionless manner. It includes Kerr-law nonlinearity and multiplicative white noise as defined by Itô, constituting a combination of three well-known models:

$$iq_t + aq_{xx} + b|q|^2 q + c_1\begin{bmatrix}\delta_1 q_{xxxx} + \delta_2 (q_x)^2 q^* + \delta_3 |q_x|^2 q + \\ \delta_4 |q|^2 q_{xx} + \delta_5 q^2 q^*_{xx} + \delta_6 |q|^4 q\end{bmatrix} + ic_2[\delta_7 q_{xxx} + \delta_8 |q|^2 q_x + \delta_9 q^2 q^*_x] + \sigma q \frac{dW(t)}{dt} = i[\alpha q_x + \lambda(|q|^2 q)_x + \beta(|q|^2)_x q]. \quad (1)$$

Here, $q(x, t)$ is a complex-valued function. The first term represents linear temporal evolution, where $i = \sqrt{-1}$. Constants a and b denote CD and Kerr-law nonlinearity, respectively. Parameters c_j for $j = 1, 2$ and δ_k for $k = 1, 2,…,9$ are constant coefficients, c_1 and c_2 are real-valued. By setting $c_1 = c_2 = 0$, we obtain the NLSE; by setting $c_2 = 0$ and $c_1 \neq 0$, we obtain the Lakshmanan–Porsezian–Daniel model; and by setting $c_1 = 0$ and $c_2 \neq 0$, we have the Sasa–Satsuma model. The model (1) illustrates soliton propagation in optical fibers, combining three known equations. The standard Wiener process $W(t)$ is included, with σ as the noise strength coefficient and $W(t)/dt$ as white noise. The coefficient for inter-modal dispersion is α, self-steepening is λ, and self-frequency shift is β.

2. *F*-expansion procedure

Consider the model equation:

$$G(q, q_x, q_t, q_{xt}, q_{xx}, \ldots) = 0. \tag{2}$$

Regarding the optoelectronic wave field $q = q(x, t)$, x is the spatial variable, and t is the temporal variable.

With account of the constraints

$$q(x,t) = U(\xi),\ \xi = \mu(x - \upsilon t). \tag{3}$$

In this context, ξ is the wave variable, μ is the wave width, and υ represents the wave velocity. As a result, Eq. (2) evolves into

$$P(U, -\mu\upsilon U', \mu U', \mu^2 U'', \ldots) = 0. \tag{4}$$

Step 1: With account of (4), the simplified model confirms the solution structure

$$U(\xi) = \sum_{i=0}^{N} B_i F^i(\xi), \tag{5}$$

with the help of the ancillary equation

$$F'(\xi) = \sqrt{PF^4(\xi) + QF^2(\xi) + R}. \tag{6}$$

Accordingly, the soliton wave profiles obtained from (6) are outlined below

$$\begin{cases} F(\xi) = \mathrm{sn}(\xi) = \tanh(\xi), P = m^2, Q = -(1+m^2), \\ \quad R = 1,\ \ m \to 1^-, \\ F(\xi) = \mathrm{ns}(\xi) = \coth(\xi), P = 1, Q = -(1+m^2), \\ \quad R = m^2,\ \ m \to 1^-, \\ F(\xi) = \mathrm{cn}(\xi) = sech\ (\xi), P = -m^2, Q = 2m^2 - 1, \\ \quad R = 1 - m^2,\ \ m \to 1^-, \\ F(\xi) = \mathrm{ds}(\xi) = csch\ (\xi),\ \ P = 1, Q = 2m^2 - 1, \\ \quad R = -m^2(1-m^2),\ \ \to 1^-, \\ F(\xi) = \mathrm{ns}(\xi) \pm \mathrm{ds}(\xi) = \coth(\xi) \pm csch\ (\xi), \\ \quad P = \frac{1}{4},\ \ Q = \frac{m^2 - 2}{2},\ \ R = \frac{m^2}{4},\ \ m \to 1^-, \\ F(\xi) = \mathrm{sn}(\xi) \pm i\mathrm{cn}(\xi) = \tanh(\xi) \pm i\ sech\ (\xi), \\ \quad P = \frac{m^2}{4},\ \ Q = \frac{m^2 - 2}{2},\ \ R = \frac{m^2}{4}, \to 1^-, \\ F(\xi) = \frac{\mathrm{sn}(\xi)}{1 \pm \mathrm{dn}(\xi)} = \frac{\tanh(\xi)}{1 \pm\ sech}, P = \frac{m^2}{4}, \\ \quad Q = \frac{m^2 - 2}{2}, R = \frac{m^2}{4},\ \ \to 1^-, \end{cases} \tag{7}$$

where sn(ξ), ns(ξ), cn(ξ), ds(ξ), and dn(ξ) denote Jacobi elliptic functions associated with a modulus, $0 < m < 1$. Additionally, the constants B_i (for i from 0 to N) stem from the balancing approach described in (4).

Step 2: Through the inclusion of (5) and (6) into (4), we create a system of equations that allows for the determination of the constants not specified in (4) via (7).

3. Optical solitons

By adopting the following solution form, we secure the solution to Eq. (1):

$$q(x,t) = U(\xi) e^{i\phi(x,t)}, \tag{8}$$

where ξ, representing the variable of the wave, is set as

$$\xi = k(x - \upsilon t). \tag{9}$$

To clarify further, υ denotes the speed of the soliton, and $U(\xi)$ represents the amplitude component. The phase component $\phi(x,t)$ is then expressed as:

$$\phi(x,t) = -\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0. \tag{10}$$

Here, κ, ω, σ, and θ_0 stand for the frequency of the solitons, wave number, noise coefficient, and phase constant, respectively. Equation (1) is rewritten by substituting (8) and then separated into its imaginary and real components, leading to

$$\begin{aligned} &k^2(a - 6c_1\delta_1\kappa^2 + 3c_2\delta_7\kappa)U'' - \\ &(a\kappa^2 + \alpha\kappa - c_1\delta_1\kappa^4 + c_2\delta_7\kappa^3 - \sigma^2 + \omega)U + \\ &c_1(\delta_4 + \delta_5)k^2U^2U'' + c_1(\delta_2 + \delta_3)k^2UU'^2 - \\ &\kappa(c_2(\delta_9 - \delta_8) + c_1(\delta_2 - \delta_3 + \delta_4 + \delta_5)\kappa + \lambda - b)U^3 + \\ &c_1\delta_6U^5 + c_1\delta_1k^4U'''' = 0, \end{aligned} \tag{11}$$

and

$$\begin{aligned} &k(-2a\kappa - \alpha + 4c_1\delta_1\kappa^3 - 3c_2\delta_7\kappa^2 - \upsilon)U' + \\ &\qquad k^3(c_2\delta_7 - 4c_1\delta_1\kappa)U''' - \\ &-k\begin{pmatrix} 2\beta - c_2(\delta_9 + \delta_8) + \\ 2c_1(\delta_2 + \delta_4 - \delta_5)\kappa + 3\lambda \end{pmatrix} U^2U' = 0. \end{aligned} \tag{12}$$

To determine the speed of solitons, we analyze the imaginary component as follows:

$$\upsilon = -2a\kappa - \alpha + 4c_1\delta_1\kappa^3 - 3c_2\delta_7\kappa^2, \tag{13}$$

with constraints on parameters

$$2\beta + 2c_1(\delta_2 + \delta_4 - \delta_5)\kappa - c_2(\delta_8 + \delta_9) + 3\lambda = 0, \tag{14}$$

and

$$c_2\delta_7 - 4c_1\delta_1\kappa = 0. \tag{15}$$

Eq. (11) reduces to

$$\begin{aligned} &k^2U^{(iv)} + A_1U^2U'' + A_2U'' + A_3UU'^2 + A_4U + \\ &A_5U^5 + A_6U^3 = 0, \end{aligned} \tag{16}$$

with

$$\begin{cases} A_1 = \frac{\delta_4 + \delta_5}{\delta_1},\ \ A_2 = \frac{a - 6c_1\delta_1\kappa^2 + 3c_2\delta_7\kappa}{c_1\delta_1}, \\ A_3 = \frac{\delta_2 + \delta_3}{\delta_1},\ \ A_4 = -\frac{a\kappa^2 + \alpha\kappa - c_1\delta_1\kappa^4 + c_2\delta_7\kappa^3 - \sigma^2 + \omega}{c_1\delta_1k^2}, \\ A_5 = \frac{\delta_6}{\delta_1k^2},\ \ A_6 = -\frac{\kappa(c_1(\delta_2 - \delta_3 + \delta_4 + \delta_5)\kappa + c_2(\delta_9 - \delta_8) + \lambda - b)}{c_1\delta_1k^2}. \end{cases} \tag{17}$$

The balancing between $U^{(iv)}$ and U^5 in Eq. (16) establishes $N = 1$, thereby expressing the solution as:

$$U(\xi) = B_0 + B_1F(\xi). \tag{18}$$

By substituting (18) with (6) into (16), we derive the equations:

$$\begin{cases} A_5B_0{}^5 + RA_3B_0B_1{}^2 + A_6B_0{}^3 + A_4B_0 = 0, \\ 12\,PRk^2B_1 + QA_1B_0{}^2B_1 + Q^2k^2B_1 + QA_2B_1 + \\ \quad RA_3B_1{}^3 + 5\,A_5B_0{}^4B_1 + 3\,A_6B_0{}^2B_1 + A_4B_1 = 0, \\ 10\,A_5B_0{}^3B_1{}^2 + 2\,QA_1B_0B_1{}^2 + QA_3B_0B_1{}^2 + \\ \qquad 3\,A_6B_0B_1{}^2 = 0, \\ 10\,A_5B_0{}^2B_1{}^3 + 20\,PQk^2B_1 + 2\,PA_1B_0{}^2B_1 + \\ \quad QA_1B_1{}^3 + QA_3B_1{}^3 + A_6B_1{}^3 + 2\,PA_2B_1 = 0, \\ 5\,A_5B_0B_1{}^4 + 4\,PA_1B_0B_1{}^2 + PA_3B_0B_1{}^2 = 0, \\ A_5B_1{}^5 + 24\,P^2k^2B_1 + 2\,PA_1B_1{}^3 + PA_3B_1{}^3 = 0. \end{cases} \tag{19}$$

The outcomes are uncovered upon solving these equations (19):

$$\begin{cases} k = \pm\sqrt{\dfrac{2\,PA_2RA_3 - Q^2A_1A_2 - Q^2A_2A_3 - QA_1A_4 - QA_2A_6 - QA_3A_4 - A_4A_6}{12\,PQRA_1 - 8\,PQRA_3 + Q^3A_1 + Q^3A_3 + 12\,PRA_6 + Q^2A_6}}, \\ A_5 = \dfrac{\begin{pmatrix} 24\,PRA_1A_2 - 12\,PA_2RA_3 - 6\,Q^2A_1A_2 + 3\,Q^2A_2A_3 \\ -8\,QA_1A_4 + 12\,QA_2A_6 + 2\,QA_3A_4 + 12\,A_4A_6 \end{pmatrix} \times \begin{pmatrix} 12\,PQRA_1 - 8\,PQRA_3 + Q^3A_1 \\ +Q^3A_3 + 12\,PRA_6 + Q^2A_6 \end{pmatrix}}{2\begin{pmatrix} 144\,P^2R^2A_2^2 - 216\,PQ^2RA_2^2 + 81\,Q^4A_2^2 - \\ 240\,PQRA_2A_4 + 180\,Q^3A_2A_4 + 100\,Q^2A_4^2 \end{pmatrix}}, \\ B_0 = 0, \\ B_1 = \pm\sqrt{-\dfrac{24\,P^2RA_2 - 18\,PQ^2A_2 - 20\,PQA_4}{12\,PQRA_1 - 8\,PQRA_3 + Q^3A_1 + Q^3A_3 + 12\,PRA_6 + Q^2A_6}}. \end{cases} \quad (20)$$

Result 1:

Based on (7), Eq. (20) changes into

$$\begin{cases} B_0 = 0, \quad B_1 = \pm\sqrt{-\dfrac{6\,A_2 - 5\,A_4}{4\,A_1 - A_3 - 2\,A_6}}, \\ k = \pm\sqrt{\dfrac{4\,A_1A_2 - 2\,A_1A_4 + 2\,A_2A_3 - 2\,A_2A_6 - 2\,A_4A_3 + A_4A_6}{32\,A_1 - 8\,A_3 - 16\,A_6}}, \\ A_5 = -\dfrac{(4\,A_1A_4 - 6\,A_2A_6 - A_4A_3 + 3\,A_4A_6)(4\,A_1 - A_3 - 2\,A_6)}{36\,A_2^2 - 60\,A_2A_4 + 25\,A_4^2}. \end{cases} \quad (21)$$

The dark and singular soliton solutions are formulated, in conclusion, as

$$q(x,t) = \pm\sqrt{-\frac{6\,A_2 - 5\,A_4}{4\,A_1 - A_3 - 2\,A_6}} \times \tanh\left[\sqrt{\frac{4\,A_1A_2 - 2\,A_1A_4 + 2\,A_2A_3 - 2\,A_2A_6 - 2\,A_4A_3 + A_4A_6}{32\,A_1 - 8\,A_3 - 16\,A_6}} \times \left(x + (2a\kappa + \alpha - 4c_1\delta_1\kappa^3 + 3c_2\delta_7\kappa^2)t\right)\right] \times e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0)}, \quad (22)$$

and

$$q(x,t) = \pm\sqrt{-\frac{6\,A_2 - 5\,A_4}{4\,A_1 - A_3 - 2\,A_6}} \times \coth\left[\sqrt{\frac{4\,A_1A_2 - 2\,A_1A_4 + 2\,A_2A_3 - 2\,A_2A_6 - 2\,A_4A_3 + A_4A_6}{32\,A_1 - 8\,A_3 - 16\,A_6}} \times \left(x + (2a\kappa + \alpha - 4c_1\delta_1\kappa^3 + 3c_2\delta_7\kappa^2)t\right)\right] \times e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0)}. \quad (23)$$

The wave forms given by equations (22) and (23) are determined by the parameter constraints:

$$(6\,A_2 - 5\,A_4)(4\,A_1 - A_3 - 2\,A_6) < 0, \quad \begin{pmatrix} 4\,A_1A_2 - 2\,A_1A_4 + 2\,A_2A_3 - \\ 2\,A_2A_6 - 2\,A_4A_3 + A_4A_6 \end{pmatrix} \times (32\,A_1 - 8\,A_3 - 16\,A_6) > 0. \quad (24)$$

Result 2:

Using (7), Eq. (20) evolves into

$$\begin{cases} B_0 = 0, \; B_1 = \pm\sqrt{-\dfrac{18\,A_2 + 20\,A_4}{A_1 + A_3 + A_6}}, \; k = \pm\sqrt{-(A_2 + A_4)}, \\ A_5 = -\dfrac{\begin{pmatrix} 6\,A_1^2A_2 + 8\,A_1^2A_4 + 3\,A_1A_2A_3 - \\ 6\,A_1A_2A_6 + 6\,A_1A_3A_4 - 4\,A_1A_4A_6 - \\ 3\,A_2A_3^2 - 15\,A_2A_3A_6 - 12\,A_2A_6^2 - \\ -2\,A_3^2A_4 - 14\,A_3A_4A_6 - 12\,A_4A_6^2 \end{pmatrix}}{2(81\,A_2^2 + 180\,A_2A_4 + 100\,A_4^2)}. \end{cases} \quad (25)$$

Thus, the form of the solution for the bright soliton is

$$q(x,t) = \pm\sqrt{-\frac{18\,A_2 + 20\,A_4}{A_1 + A_3 + A_6}} \times \operatorname{sech}\left[\sqrt{-(A_2 + A_4)} \times \left(x + (2a\kappa + \alpha - 4c_1\delta_1\kappa^3 + 3c_2\delta_7\kappa^2)t\right)\right] \times e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0)}. \quad (26)$$

The wave form described in (26) is governed by the constraint relations:

$$(18\,A_2 + 20\,A_4)(A_1 + A_3 + A_6) < 0, \quad A_2 + A_4 < 0. \quad (27)$$

Result 3:

With (7) being employed, Eq. (20) is changed to

$$\begin{cases} B_0 = 0, \quad B_1 = \pm\sqrt{\dfrac{18\,A_2 + 20\,A_4}{A_1 + A_3 + A_6}}, \quad k = \pm\sqrt{-(A_2 + A_4)}, \\ A_5 = -\dfrac{\begin{pmatrix} 6\,A_1^2A_2 + 8\,A_1^2A_4 + 3\,A_1A_2A_3 - \\ 6\,A_1A_2A_6 + 6\,A_1A_3A_4 - 4\,A_1A_4A_6 - \\ 3\,A_2A_3^2 - 15\,A_2A_3A_6 - 12\,A_2A_6^2 - \\ 2\,A_3^2A_4 - 14\,A_3A_4A_6 - 12\,A_4A_6^2 \end{pmatrix}}{2(81\,A_2^2 + 180\,A_2A_4 + 100\,A_4^2)}. \end{cases} \quad (28)$$

Therefore, the solution representing the singular soliton is

$$q(x,t) = \pm\sqrt{\frac{18\,A_2 + 20\,A_4}{A_1 + A_3 + A_6}} \times \operatorname{csch}\left[\sqrt{-(A_2 + A_4)} \times \left(x + (2a\kappa + \alpha - 4c_1\delta_1\kappa^3 + 3c_2\delta_7\kappa^2)t\right)\right] \times e^{i(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0)}. \quad (29)$$

The wave form depicted in (29) is determined by the constraint relations:

$$(18\,A_2 + 20\,A_4)(A_1 + A_3 + A_6) > 0, \quad A_2 + A_4 < 0. \quad (30)$$

Result 4:

Applying (7), Eq. (20) takes the form

$$\begin{cases} B_0 = 0, \quad B_1 = \pm\sqrt{-\dfrac{6\,A_2 - 20\,A_4}{4\,A_1 - A_3 - 8\,A_6}}, \\ k = \pm\sqrt{\dfrac{2\,A_1A_2 - 4\,A_1A_4 + A_2A_3 - 4\,A_2A_6 - 4\,A_4A_3 + 8\,A_4A_6}{4\,A_1 - A_3 - 8\,A_6}}, \\ A_5 = -\dfrac{(4\,A_1A_4 - 6\,A_2A_6 - A_4A_3 + 12\,A_4A_6)(4\,A_1 - A_3 - 8\,A_6)}{4(9\,A_2^2 - 60\,A_2A_4 + 100\,A_4^2)}. \end{cases} \quad (31)$$

The solution for the straddled singular-singular soliton in this case is

$$q(x,t) = \pm\sqrt{-\frac{6A_2-20A_4}{4A_1-A_3-8A_6}}$$

$$\times\left\{\begin{array}{l}\coth\left[\sqrt{\frac{2A_1A_2-4A_1A_4+A_2A_3-4A_2A_6-4A_4A_3+8A_4A_6}{4A_1-A_3-8A_6}}\times\left(x+\left(2\alpha\kappa+\alpha-4c_1\delta_1\kappa^3+3c_2\delta_7\kappa^2\right)t\right)\right]\\ \pm\operatorname{csch}\left[\sqrt{\frac{2A_1A_2-4A_1A_4+A_2A_3-4A_2A_6-4A_4A_3+8A_4A_6}{4A_1-A_3-8A_6}}\times\left(x+\left(2\alpha\kappa+\alpha-4c_1\delta_1\kappa^3+3c_2\delta_7\kappa^2\right)t\right)\right]\end{array}\right\}$$

$$\times e^{i\left(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \qquad (32)$$

In addition, the representation of the complexiton solution is

$$q(x,t) = \pm\sqrt{-\frac{6A_2-20A_4}{4A_1-A_3-8A_6}}\times$$

$$\left\{\begin{array}{l}\tanh\left[\sqrt{\frac{2A_1A_2-4A_1A_4+A_2A_3-4A_2A_6-4A_4A_3+8A_4A_6}{4A_1-A_3-8A_6}}\times\left(x+\left(2\alpha\kappa+\alpha-4c_1\delta_1\kappa^3+3c_2\delta_7\kappa^2\right)t\right)\right]\\ \pm i\operatorname{sech}\left[\sqrt{\frac{2A_1A_2-4A_1A_4+A_2A_3-4A_2A_6-4A_4A_3+8A_4A_6}{4A_1-A_3-8A_6}}\times\left(x+\left(2\alpha\kappa+\alpha-4c_1\delta_1\kappa^3+3c_2\delta_7\kappa^2\right)t\right)\right]\end{array}\right\}\times$$

$$\times e^{i\left(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \qquad (33)$$

Also, the solution for the straddled dark-bright soliton is

$$q(x,t) = \pm\sqrt{-\frac{6A_2-20A_4}{4A_1-A_3-8A_6}}$$

$$\times\left\{\frac{\tanh\left[\sqrt{\frac{2A_1A_2-4A_1A_4+A_2A_3-4A_2A_6-4A_4A_3+8A_4A_6}{4A_1-A_3-8A_6}}\times\left(x+\left(2\alpha\kappa+\alpha-4c_1\delta_1\kappa^3+3c_2\delta_7\kappa^2\right)t\right)\right]}{1\pm\operatorname{sech}\left[\sqrt{\frac{2A_1A_2-4A_1A_4+A_2A_3-4A_2A_6-4A_4A_3+8A_4A_6}{4A_1-A_3-8A_6}}\times\left(x+\left(2\alpha\kappa+\alpha-4c_1\delta_1\kappa^3+3c_2\delta_7\kappa^2\right)t\right)\right]}\right\} \qquad (34)$$

$$\times e^{i\left(-\kappa x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}.$$

The wave forms described in (32)–(34) adhere to the parametric restrictions:

$$(6A_2-20A_4)(4A_1-A_3-8A_6)<0,$$
$$\left(\begin{array}{c}2A_1A_2-4A_1A_4+A_2A_3-4A_2A_6\\ -4A_4A_3+8A_4A_6\end{array}\right)\times$$
$$\times(4A_1-A_3-8A_6)>0. \qquad (35)$$

4. Conclusions

This paper investigates the concatenation model in the presence of white noise. An integration approach, namely the *F*-expansion method, is employed to reveal soliton solutions for the model. Soliton solutions are obtained by choosing specific parameter values. Additionally, this approach provides a comprehensive spectrum of optical soliton solutions. A novel observation in this paper is that for the concatenation model, the effect of white noise is confined to the phase component of the solitons. The findings of this study have broad implications. Future work will extend the model to include differential group delay and dispersion-flattened fibers, where white noise components will be introduced and analyzed using various integration approaches. This exploration is expected to yield compelling results that could significantly advance the field of mathematical photonics. Ongoing research activities will continue to unfold and disseminate these novel findings, aligning them with existing literature.

Disclosure

The authors claim there is no conflict of interest.

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Оптичне збурення солітонів з моделлю конкатенації, що має мультиплікативний білий шум за *F*-розширенням

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Анотація. У статті досліджується модель конкатенації, що включає закон Керра фазової самомодуляції та гамільтонові збурення, а також вплив мультиплікативного білого шуму. Для отримання розв'язків моделі реалізовано алгоритм *F*-розширення, що дає повний спектр оптичних солітонів. Також перераховано параметричні обмеження, які природно випливають зі схеми.

Ключові слова: солітони, білий шум, *F*-розширення.