Semiconductor Physics, Quantum Electronics & Optoelectronics, 2009. V. 12, N 4. P. 349-356.

PACS 61.72, 72.20

Electron-electron drag in crystals with a multi-valley band. Magnetoresistivity and Hall-effect

I.I. Boiko
V. Lashkaryov Institute of Semiconductor Physics, NAS of Ukraine,
45, prospect Nauky, 03028 Kyiv, Ukraine, e-mail: igorboiko@yandex.ru, phone: (044)236-5422

Abstract. Hall-effect and magnetoresistivity of electrons in multi-valley bands of Si and Ge is considered with due regard for direct intervalley drag. Search of contribution of this drag shows that this interactioin sufficiently changes both effects. Calculated here values substantially differ from consequent those obtained on the base of popular τ-approximation.

Keywords: quantum kinetic equation, Hall-effect, mobility, intervalley drag.

Manuscript received 06.07.09; accepted for publication 10.09.09; published online 30.10.09.

1. Introduction

It was shown in the previous paper (see Ref. [1]) that *e-e*-interaction in multi-valley semiconductor with equivalent anisotropic valleys (germanium and silicon, for example) introduces specific contribution to conductivity. The intervalley drag cannot principally be considered in framework grounded on popular τ-approximation (see Refs [2–5]). Results obtained in the paper [1] give a sufficient basis to expect appearance of conformable new results to other kinetic phenomena. Specific interest consists in comparison of some values (Hall-constant and magnetoresistivity), calculated by the proposed method of balance equation, with analogous values obtained using the of ordinary method of scalar relaxation time.

2. Quantum kinetic equation

Consider here an uniform crystal in constant uniform electrical and magnetic fields $\vec{E}$ and $\vec{H}$. For this case the stationary quantum kinetic equation for nonequilibrium distribution function $f_{\vec{k}}^{(a)}$ of carriers from *a*-valley can be presented in the following form (see Refs [1, 6 – 8]):

$$\frac{e_a}{\hbar}\vec{E}\frac{\partial f_{\vec{k}}^{(a)}}{\partial\vec{k}}+\frac{e_a}{\hbar}\left\{\frac{1}{c}\left[\left(\vec{H}\times\frac{\partial}{\partial\vec{k}}\right),\vec{v}^{(a)}(\vec{k})\right]_+ f_{\vec{k}}^{(a)}\right\}=$$

$$=St f_{\vec{k}}^{(a)}=St_{e-I}f_{\vec{k}}^{(a)}+St_{e-ph}f_{\vec{k}}^{(a)}+\sum_{b=1}^{N}St_{a-b}f_{\vec{k}}^{(a)}$$

$(a, b = 1, 2, \ldots, N)$. (1)

Here the total number of valleys $N = 6$ for *n*-Si and $N = 4$ for *n*-Ge (see Fig. 1). The vector

$$\vec{v}^{(a)}(\vec{k})=\hbar^{-1}(\partial\varepsilon_{\vec{k}}^{(a)}/\partial\vec{k})$$

is the microscopic velocity of *a*-carriers and $\varepsilon_{\vec{k}}^{(a)}$ is their dispersion law. The right part of Eq. (1) is the collision integral for *a*-carriers. In this paper, we take into consideration interaction of band electrons with uniformly distributed charged impurities, equilibrium longitudinal acoustic phonons and carriers belonging to all valleys in the conduction band.

3. Balance equation and kinetic coefficients

Applying to both sides of Eq. (1) the operator

$$(1/4\pi^3)\int\vec{k}d^3\vec{k},$$

one obtains a set of exact balance equations for dynamic and statistic forces:

$$e_a[\vec{E}+(1/c)(\vec{H}\times\vec{u}^{(a)})]+\vec{C}^{(a)}(\vec{H})-$$
$$-\vec{\nabla}\varepsilon_F^{(a)}(\vec{r})+\vec{F}_T^{(a)}+\vec{F}_{e-I}^{(a)}+\vec{F}_{e-ph}^{(a)}+\sum_b\vec{F}^{(a,b)}=0 \qquad (2)$$

Here (see [1, 7])

$$\vec{u}^{(a)}=\int\vec{v}^{(a)}(\vec{k})f_{\vec{k}}^{(a)}d^3\vec{k}\Big/\int f_{\vec{k}}^{(a)}d^3\vec{k}$$

is the drift velocity of particles from *a*-group,

$$\vec{F}_{e-I}^{(a)}+\vec{F}_{e-ph}^{(a)}=-\frac{e^2}{(2\pi)^6 n_a}\int d^3\vec{k}\int\vec{q}d^3\vec{q}\times$$
$$\times\int d\omega\,\delta(\hbar\omega-\varepsilon_{\vec{k}}^{(a)}+\varepsilon_{\vec{k}-\vec{q}}^{(a)})\{f_{\vec{k}}^{(a)}-f_{\vec{k}-\vec{q}}^{(a)}+[f_{\vec{k}}^{(a)}(1-f_{\vec{k}-\vec{q}}^{(a)})+$$
$$+f_{\vec{k}-\vec{q}}^{(a)}(1-f_{\vec{k}}^{(a)})]\tanh(\hbar\omega/2k_BT)\}\times$$
$$\times[\langle\varphi_{(I)}^2\rangle_{\omega,\vec{q}}+\langle\varphi_{(ph)}^2\rangle_{\omega,\vec{q}}]; \qquad (3)$$

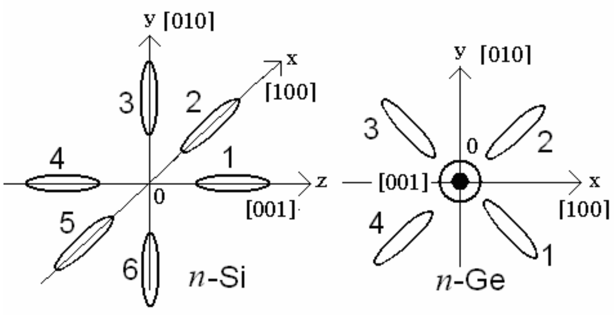

Fig.1

$$\vec F^{(a,b)} = \frac{e^4\hbar}{4\pi^6 n^{(a)}}\int \vec k\, d^3\vec k \int d^3\vec k' \times$$

$$\times \int d^3\vec q\, \frac{1}{q^4}\frac{\delta(\varepsilon^{(a)}_{\vec k} - \varepsilon^{(a)}_{\vec k-\vec q} - \varepsilon^{(b)}_{\vec k'} + \varepsilon^{(b)}_{\vec k'-\vec q'})}{\left|\varepsilon_L + \Delta\varepsilon(\omega=0,\vec q)\right|^2} D_{ab}(\vec k,\vec k',\vec q)\,;$$

$$D_{ab}(\vec k,\vec k',\vec q) = f^{(a)}_{\vec k-\vec q}(1-f^{(a)}_{\vec k}) f^{(b)}_{\vec k'}(1-f^{(b)}_{\vec k'-\vec q}) -$$

$$- f^{(a)}_{\vec k}(1-f^{(a)}_{\vec k-\vec q}) f^{(b)}_{\vec k'-\vec q}(1-f^{(b)}_{\vec k'})\,. \qquad (4)$$

For n-Si and n-Ge the term $\vec C^{(a)}(\vec H)$ equals to zero. $n^{(a)}$ is the density of electrons from a-group. Here $n^{(a)} = n/N$.

Our next step is the choice of appropriate models of non-equilibrium distribution functions $f^{(a)}_{\vec k}$. As a model for a-group we accept the Fermi function with an argument containing the shift of velocity $\vec v^{(a)}(\vec k) = \hbar^{-1}(\partial\varepsilon^{(a)}_{\vec k}/\partial\vec k)$ by the correspondent drift velocity $\vec u^{(a)}$ for the whole a-group:

$$f^{(a)}_{\vec k} = f^{0(a)}(\vec v^{(a)}(\vec k) - \vec u^{(a)})\,. \qquad (5)$$

Here,

$$f^{0(a)}(\vec v^{(a)}(\vec k)) = f_0\,(\varepsilon^{(a)}_{\vec k}) = [1+\exp(\varepsilon^{(a)}_{\vec k} - \varepsilon_F)/k_BT]^{-1}$$

is the equilibrium distribution function for a-carriers and $\varepsilon^{(a)}_{\vec k}$ is their dispersion law.

Using the form (5) and carrying out linearization of forces in Eqs (3) and (4), we obtain:

$$\vec F^{(a)}_{e-I} + \vec F^{(a)}_{e-ph} = -e\ \tilde\beta^{(a)}\vec u^{(a)}\ ;$$

$$\vec F^{(a,b)} = -e\ \tilde\xi^{(a,b)}\left(\vec u^{(a)} - \vec u^{(b)}\right). \qquad (6)$$

Then the set of balance equations (2) takes the form

$$\vec E = \sum_b \tilde Y^{(a,b)}(\vec G)\vec u^{(b)} \qquad (a = 1, 2, \ldots, N)\,, \qquad (7)$$

where

$$\tilde Y^{(a,b)}(\vec G) = \beta_\perp[\tilde V^{(a)} + \beta_\perp^{-1}\sum_m \tilde\xi^{(a,m)} - \tilde\eta(\vec G)])\delta_{ab} - \tilde\xi^{(a,b)}$$

$$(a,\, b,\, m\ = 1, 2, \ldots, N)\,, \qquad (8)$$

$$\vec G = (1/c\beta_\perp)\vec H\ , \quad \tilde\eta(\vec G) = \begin{pmatrix} 0 & -G_z & G_y \\ G_z & 0 & -G_x \\ -G_y & G_x & 0 \end{pmatrix}\ . \qquad (9)$$

Matrices $\tilde V^{(a)}$ for n-Si and n-Ge will be shown below.

For simplicity of calculations we assume

$$\xi^{(1,2)}_{ij} = \xi^{(1,3)}_{ij} = \xi^{(2,3)}_{ij} = \delta_{ij}\xi \qquad (10)$$

and use the following simplified forms (see Refs [1, 7]):

$$\beta_\perp = \beta^{(I)}_\perp + \beta^{(Ac)}_\perp\,;$$

$$\beta^{(I)}_\perp = \frac{8Ne^3 n_I m_\parallel^2}{3\pi\hbar^3\varepsilon_L^2\, nL(L+1)^2}\int_0^\infty \frac{w\,dw}{1+\exp(w-\eta)}\frac{1}{[w+w_{scr}(w)]^2}\,; \qquad (11)$$

$$\beta^{(Ac)}_\perp = \frac{8N\Xi_A^2(k_BT)^3 m_\parallel^4}{3\pi^3 e\hbar^7\rho s^2 nL^2}\int_0^\infty \frac{w^3 dw}{1+\exp(w-\eta)}\frac{1}{[w+w_{scr}(w)]^2}\,; \qquad (12)$$

$$\xi = \frac{\sqrt2 e^3(k_BT)^{3/2}\,\varpi}{\pi^3 n\ \hbar^6\varepsilon_L^2}\left(\frac{3m_\parallel}{2L+1}\right)^{7/2} \times$$

$$\times \int_0^\infty \frac{\sqrt w\, dw}{\left[1+\exp(w-\eta)\right]^2}\frac{1}{[w+w_{scr}(w)]^2}\,; \qquad (13)$$

$$w_{scr}(w) = \frac{e^2N(2L+1)\sqrt{2m_\parallel}}{8\pi L\varepsilon_L\ \hbar\sqrt{w\,k_BT}} \times$$

$$\times \int_0^\infty \frac{d\varepsilon}{1+\exp(\varepsilon-\eta)}\ln\left(\left|\frac{\sqrt\varepsilon+\sqrt w}{\sqrt\varepsilon-\sqrt w}\right|\right)\,; \qquad (14)$$

$$\varpi = \int_{-\infty}^{\infty}\frac{\chi^2 d\chi}{\sinh^2\chi} \approx 3{,}29\ ; \quad \eta = \frac{\varepsilon_F}{k_BT}\ . \qquad (15)$$

Farther we use the following designations:

$$\zeta = N\xi/\beta_\perp\ , \quad (1/N)\sum_a \vec u^{(a)} = \vec u\ . \qquad (16)$$

The total density of current:

$$\vec j = \sum_{a=1}^{N}\vec j^{(a)} = e\sum_{a=1}^{N} n^{(a)}\vec u^{(a)} = e\sum_{a=1}^{N}(n/N)\vec u^{(a)} = en\vec u\ . \qquad (17)$$

Now, Eq.(7) can be presented in the form

$$(1/\beta_\perp)\vec E = \tilde C^{(a)}(\zeta,\,\vec G)\vec u^{(a)} - \zeta\,\vec u\ , \qquad (18)$$

where

$$\tilde C^{(a)}(\zeta,\,\vec G) = \tilde V^{(a)} + \zeta\,\tilde I - \tilde\eta(\vec G)\,. \qquad (19)$$

It follows from Eqs (19), (20):

$$\vec u^{(a)} = [\tilde C^{(a)}]^{-1}[(1/\beta_\perp)\vec E + \zeta\,\vec u]\,;$$

$$\vec{u} = N^{-1}\{\sum_a [\tilde{C}^{(a)}(\zeta, \vec{G})]^{-1}\}[(1/\beta_\perp)\vec{E} + \zeta\,\vec{u}] \;. \tag{20}$$

As a result one obtains:

$$\vec{j} = en\tilde{\mu}(\xi, \vec{H})\vec{E} = \tilde{\sigma}(\xi, \vec{H})\vec{E}\,, \tag{21}$$

where

$$\tilde{\mu}(\xi, \vec{H}) = (1/\beta_\perp)\,\tilde{M}(\xi, \vec{H})\,; \quad \tilde{M}(\xi, \vec{H}) = (\tilde{I} - \zeta\tilde{S}(\zeta, G))^{-1}\tilde{S}(\zeta, \vec{G})\,; \tag{22}$$

$$\tilde{S}(\zeta, \vec{G}) = (1/N)\sum_{a=1}^{N}[\tilde{C}^{(a)}(\zeta, \vec{G})]^{-1}\,. \tag{23}$$

Consider farther the case

$$\vec{H} = (H_x, 0, 0)\;,\ \text{or}\ \vec{G} = (G, 0, 0)\,,\ \text{and}\ \vec{j} = (0, 0, j_z)\,. \tag{24}$$

Then

$$j_z(\xi, \vec{H}) = \sigma_H(\xi, \vec{H})E_z\,;\ \sigma_H(\xi, H) = (e\,n/\beta_\perp)(M_{zz}^2(\zeta, G) + M_{zy}^2(\zeta, G))/M_{zz}(\zeta, G)\,. \tag{25}$$

Introduce the Hall constant R_H by the relation

$$R_H(\xi, H) = E_\perp / H\,j_z = (1/enc)\,\frac{G^{-1}M_{zy}(\zeta, G)}{M_{zy}^2(\zeta, G) + M_{zz}^2(\zeta, G)}\,. \tag{26}$$

It follows from Eqs (20), (22) – (26):

$$R_H(\xi = 0, H \to 0) = \frac{1}{enc}\;. \tag{27}$$

Note that the value (27) does not depend on any specifics of external scattering system. This result does not also require to use a specific model for the nonequilibrium distribution function; it is the sequence only of approach of linear approximation over the drift velocity.

4. Galvanomagnetic effects in *n*-Si

For six valleys of *n*-Si (see Fig. 1)

$$\tilde{V}^{(1)} = \tilde{V}^{(4)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & L \end{pmatrix}\;;\quad \tilde{V}^{(2)} = \tilde{V}^{(5)} = \begin{pmatrix} L & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\;;\quad \tilde{V}^{(3)} = \tilde{V}^{(6)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & 1 \end{pmatrix}\;, \tag{28}$$

where $L = m_\parallel / m_\perp$.

One obtains from Eqs (20) and (23):

$$\tilde{S}(\zeta, G) = \begin{pmatrix} (3q+2p)/3q(q+p) & 0 & 0 \\ 0 & \dfrac{2G^2(q+p)+q^2(3q+2p)}{3(G^2+qp+q^2)(G^2+q^2)} & -\dfrac{G[3(G^2+q^2)+qp]}{3(G^2+q^2)(G^2+qp+q^2)} \\ 0 & \dfrac{G[3(G^2+q^2)+qp]}{3(G^2+q^2)(G^2+qp+q^2)} & \dfrac{2G^2(q+p)+q^2(3q+2p)}{3(G^2+qp+q^2)(G^2+q^2)} \end{pmatrix}; \tag{29}$$

here $q = 1 + \zeta$; $p = L - 1$. (30)

It follows from Eqs (22) and (29):

$$M_{xx} = (3q + 2p)/(3q + 2p + q\,p)\,; \quad M_{xy} = M_{yx} = M_{xz} = M_{zx} = 0\;;$$

$$M_{yy} = M_{zz} = \frac{u(p,q,G)[1-(q-1)u(p,q,G)] + (q-1)[v(p,q,G)]^2G^2}{(q-1)^2[v(p,q,G)]^2G^2 + [1-(q-1)u(p,q,G)]^2}\;; \tag{31}$$

$$M_{zy} = -M_{yz} = \frac{v(p,q,G)\,G}{(q-1)^2[v(p,q,G)]^2G^2 + [1-(q-1)u(p,q,G)]^2}\,;$$

$$u(p,q,G)=\frac{2(p+q)G^2+q^2(2p+3q)}{3(G^2+q^2)(G^2+q^2+pq)} \; ;$$

$$v(p,q,G)=\frac{3G^2+q(p+3q)}{3(G^2+q^2)(G^2+q^2+pq)} \, . \qquad (32)$$

One can see from expressions (26), (31), (32) that the Hall constant R_H at $H \to 0$ depends only on two ratios: $m_\parallel / m_\perp$ and $\xi/\beta_\perp$. Obtained here formulae show also that for negligible *e-e*-drag ($q \to 1$) the value R_H in contrary to the standard result (see Ref. [3]) depends only on the ratio $L = m_\parallel / m_\perp$. The same results will be obtained below for *n*-germanium.

As an external scattering system, we consider here equilibrium longitudinal acoustic phonons and charged impurities with the density n_I. In the course of numerical calculation of the value $\beta_\perp$ we assume $n = n_I$ and $\Xi_A = \Xi_d + (1/2)\Xi_u$. Here Ξ_d and Ξ_u are dilation and shear deformation potential constants. In accordance with experimental data from Ref. [8] $\Xi_A = [5.0 + (1/2)\cdot 8.8]\text{eV} = 9.4\text{eV}$ for *n*-Si and $\Xi_A = [-12.3 + (1/2)\cdot 16.3]\ \text{eV} = -4.2\ \text{eV}$ for *n*-Ge.

Fig. 2 shows calculated dependences of the Hall constant R_H and longitudinal conductivity σ_H on the intensity of external magnetic field H for band electrons in silicon. Here $H_0 = cm_0/e\tau_0$. The ratio $H/H_0 = 1$ for magnetic field B=5.69 Tl (we assume $m_0 = 9.1\times 10^{-28}\,\text{g}$, $\tau_0 = 10^{-12}\,s$). Then (see above Eq. (9)) $G = \chi H/H_0$, where $\chi = (e\tau_0\beta_\perp/m_0)^{-1}$. In this figure and farther $\sigma_\perp = en/\beta_\perp$; curves *1* relate to the case $\xi \neq 0$ (intervalley drag was involved in consideration), curves *2* – $\xi = 0$ (intervalley drag is out of consideration).

One can see from Fig. 2 that at small magnetic field ($H << H_0$)

$$R_H(\xi \neq 0) > R_H(\xi = 0)\ ,\ \ \sigma_H(\xi \neq 0) < \sigma_H(\xi = 0)\, .$$

At moderate and high magnetic fields one obtains the contrary relations:

$$R_H(\xi \neq 0) < R_H(\xi = 0)\ ,\ \ \sigma_H(\xi \neq 0) > \sigma_H(\xi = 0)\, .$$

In Fig. 3 we show calculated dependences of $enc\,R_H$ on the dimensionless Fermi energy $\eta = \varepsilon_F / k_B T$ for the limit case $H \to 0$. Here stroke lines $W^{(Si)}$ relate to the case $\xi = 0$ (intervalley drag is excluded from consideration). We used here the expression following from Eq. (26):

$$W = enc\,R_H(\xi = 0, H \to 0) = \\ = \frac{1}{M_{zz}^2(0,0)}\left[G^{-1} M_{zy}(0,0)\right]_{G\to 0} \, . \qquad (33)$$

It follows from Eqs (33) and (29):

$$W^{(Si)} = \frac{3(3+p)(p+1)}{(3+2p)^2} = \frac{3L(2+L)}{(1+2L)^2} = 0.871\, . \qquad (34)$$

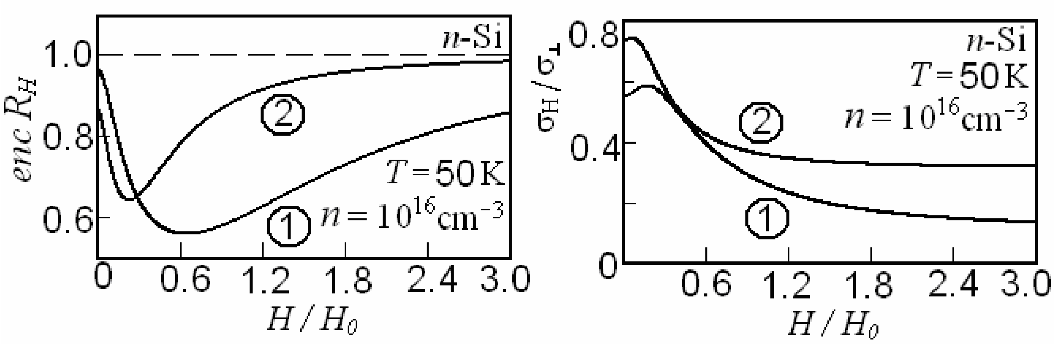

Fig. 2.

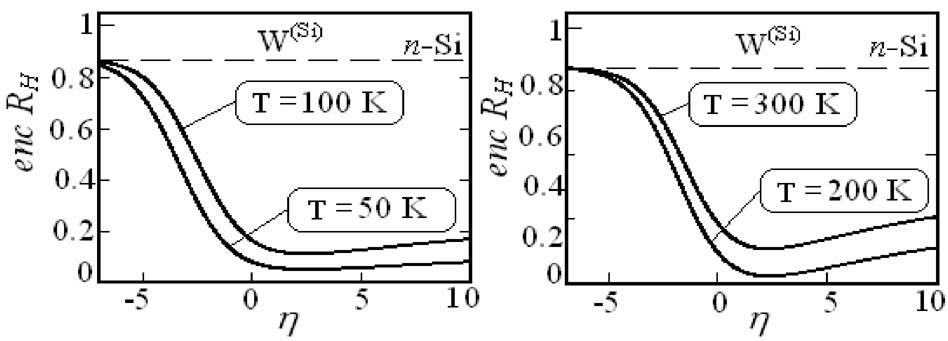

Fig. 3.

Note that the expression (34) contrary to the standard result (see Ref. [3]), does not contain characteristics of an external scattering system and can be considered as an exact relation (for the adopted above limit of linear approximation of terms in balance equation).

One can see from Fig. 3 that intervalley drag introduces negligible contribution, when $\eta < -7$.

5. Galvanomagnetic effects in *n*-Ge

For four valleys of *n*-Ge (see Fig.1) the matrices $\tilde{V}^{(a)}$ in Eq. (8) have the following forms:

$$\tilde{V}^{(1)} = \frac{1}{3}\begin{pmatrix} 2L+1 & L-1 & 1-L \\ L-1 & 2L+1 & L-1 \\ 1-L & L-1 & 2L+1 \end{pmatrix};$$

$$\tilde{V}^{(2)} = \frac{1}{3}\begin{pmatrix} 2L+1 & 1-L & 1-L \\ 1-L & 2L+1 & 1-L \\ 1-L & 1-L & 2L+1 \end{pmatrix};$$

$$\tilde{V}^{(3)} = \frac{1}{3}\begin{pmatrix} 2L+1 & L-1 & L-1 \\ L-1 & 2L+1 & 1-L \\ L-1 & 1-L & 2L+1 \end{pmatrix};$$

$$\tilde{V}^{(4)} = \frac{1}{3}\begin{pmatrix} 2L+1 & 1-L & L-1 \\ 1-L & 2L+1 & L-1 \\ L-1 & L-1 & 2L+1 \end{pmatrix}. \quad 35)$$

It follows from Eqs (20):

$$\tilde{C}^{(1)} = \begin{pmatrix} q+2r & r & -r \\ r & q+2r & G+r \\ -r & -G+r & q+2r \end{pmatrix};$$

$$\tilde{C}^{(2)} = \begin{pmatrix} q+2r & -r & -r \\ -r & q+2r & G-r \\ -r & -G-r & q+2r \end{pmatrix};$$

$$\tilde{C}^{(3)} = \begin{pmatrix} q+2r & r & r \\ r & q+2r & G-r \\ r & -G-r & q+2r \end{pmatrix}; \quad (36)$$

$$\tilde{C}^{(4)} = \begin{pmatrix} q+2r & -r & r \\ -r & q+2r & G+r \\ r & -G+r & q+2r \end{pmatrix}.$$

Here, $q = 1+\zeta$, $r = (L-1)/3$. Then, in accordance with Eqs (23) and (36),

$$\tilde{S}(\zeta,G) = \begin{pmatrix} \frac{G^2+3r^2+4rq+q^2}{G^2(q+2r)+q(q+3r)^2} & 0 & 0 \\ 0 & \frac{3r^2+4rq+q^2}{G^2(q+2r)+q(q+3r)^2} & -\frac{G(q+2r)}{G^2(q+2r)+q(q+3r)^2} \\ 0 & \frac{G(q+2r)}{G^2(q+2r)+q(q+3r)^2} & \frac{3r^2+4rq+q^2}{G^2(q+2r)+q(q+3r)^2} \end{pmatrix} \quad (37)$$

Introducing this matrix $\tilde{S}$ to Eq. (22), we find the matrix $M(r,q,G)$:

$$M_{xx} = \frac{G^2+3r^2+4rk+k^2}{(1+2r)G^2+k^2(1+2r)+2rk(2+3r)+3r^2};$$

$$M_{xy} = M_{yx} = M_{xz} = M_{zx}(r,k,G) = 0;$$

$$M_{yy} = M_{zz} = \frac{b(r,q,G)[1-(q-1)b(r,q,G)]+(q-1)[k(r,q,G)]^2G^2}{[G(q-1)k(r,q,G)]^2+[1-(q-1)b(r,q,G)]^2};$$

$$M_{yz} = -M_{zy} = \frac{G\,k(r,q,G)}{[G(q-1)k(r,q,G)]^2+[1-(q-1)b(r,q,G)]^2}; \quad (38)$$

here,

$$k(r,q,G) = \frac{2r+q}{G^2(q+2r)+q(q+3r)^2},$$

$$b(r,q,G) = \frac{3r^2+4pq+q^2}{G^2(q+2r)+q(q+3r)^2}.$$

Fig. 4 relates to band electrons in germanium and presents dependences of the Hall constant R_H and longitudinal conductivity σ_H (see Eqs (25) and (26)) on the intensity of magnetic field H.

Fig. 4 shows that at low magnetic field ($H << H_0$)

$R_H(\xi \neq 0) > R_H(\xi = 0)$;

at moderate and high magnetic fields, we have the contrary relations:

$R_H(\xi \neq 0) < R_H(\xi = 0)$.

For an arbitrary intensity of magnetic field

$\sigma_H(\xi \neq 0) < \sigma_H(\xi = 0)$.

It is worthy to note that intervalley drag manifests itself in germanium better than in silicon. It is related with higher anisotropy of germanium valleys.

Fig. 5 shows calculated for *n*-Ge dependences of $enc\,R_H$ on the dimensionless Fermi energy for the limit case $H \to 0$. Here stroke lines $W^{(\mathrm{Ge})}$ relate to the formal case $\xi = 0$ (intervalley drag is excluded from consideration). For *n*-Ge

$$W^{(\mathrm{Ge})} = \frac{(1+2r)(1+3r)^2}{(3r^2+4r+1)^2} = \frac{3(1+2L)}{(2+L)^2} = 0.261. \quad (39)$$

Note again that expression (39), as the expression (34), in difference of standard result (see Ref. [3]) does not contain characteristics of external scattering system and can be considered for adopted above limit of linear approximation for balance equations as exact relation. Intervalley drag is not actual at $\eta < -7$.

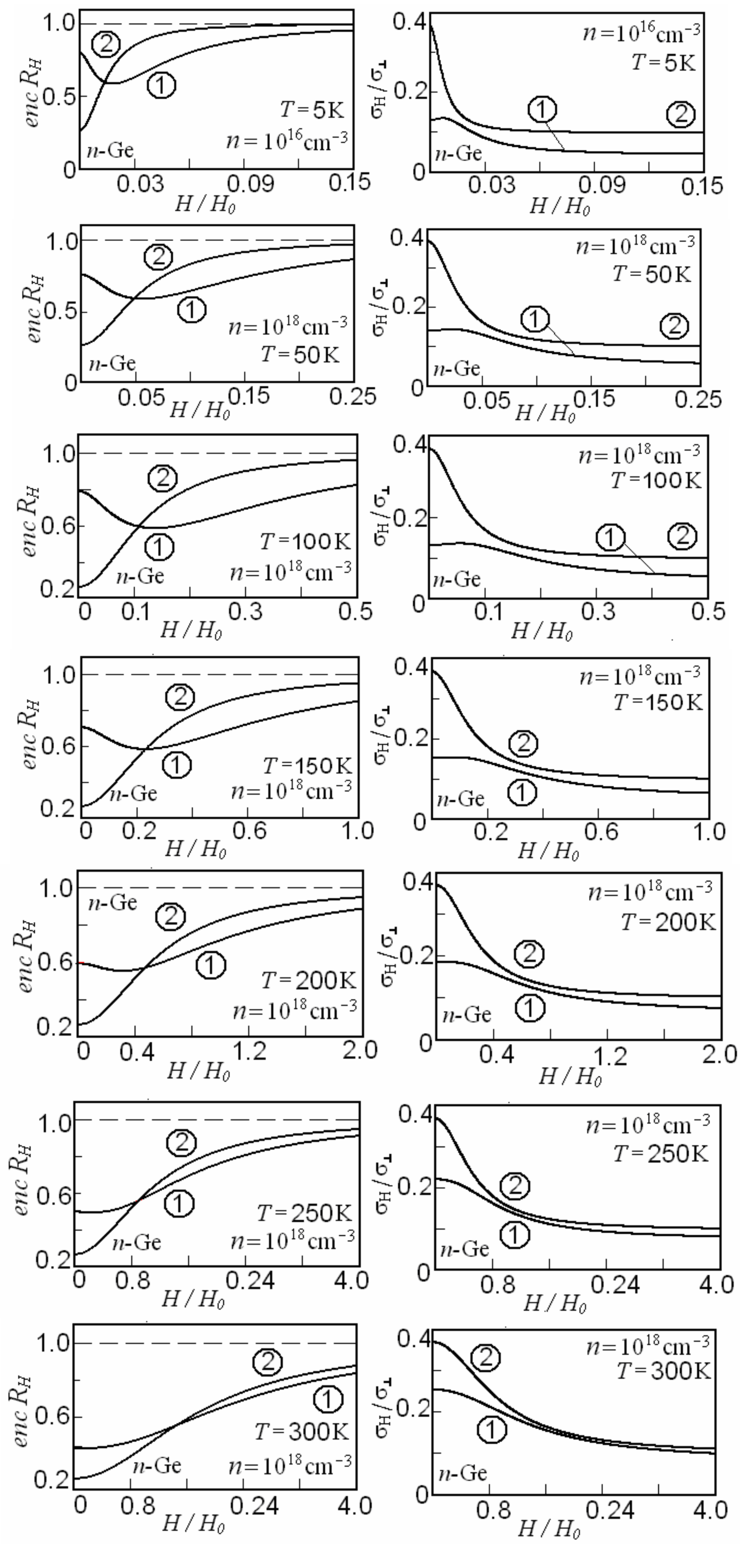

Fig. 4.

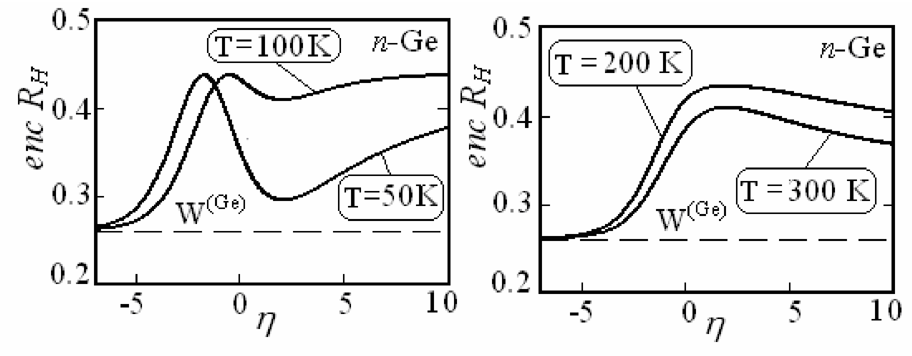

Fig. 5.

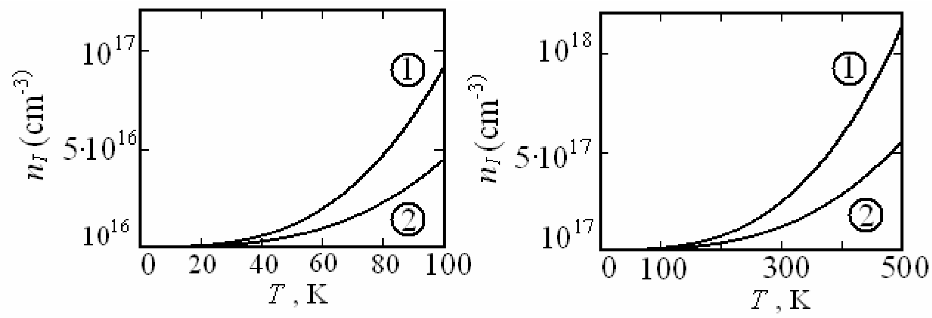

Fig. 6.

6. Calculation of the Hall-constant in *τ*-approximation

Now we present results of investigation of the Hall-effect in electron germanium and silicon obtained using the traditional τ-approximation for collision integral in kinetic equation (see for instance Ref. [3]). The well-known result for lowl magnetic field has the following form:

$$enc\, R_H^{(A)}(H \to 0) = \Lambda(L)\frac{\langle\tau^2\rangle}{\langle\tau\rangle^2}, \quad 40)$$

where $\tau\ (\varepsilon) = \tau_0 \times (\varepsilon / k_B T)^r$, angle brackets represent averaging over the energy and

$$\Lambda(L) = \frac{3L(L+2)}{(1+2L)^2}. \quad (41)$$

Carrying out averaging proposed in [3], one obtains:

$$\frac{\langle\tau^2\rangle}{\langle\tau\rangle^2} = \frac{\Gamma(2r+5/2)\Gamma(5/2)}{[\Gamma(r+5/2)]^2}\frac{K_{2r+1/2}(\eta)K_{1/2}(\eta)}{[K_{r+1/2}(\eta)]^2}.$$

Here Γ is gamma-function, and

$$K_c(\eta) = \int_0^\infty \frac{\varepsilon^c\, d\varepsilon}{1+\exp(\varepsilon-\eta)}.$$

In *n*-Si $\Lambda(L) = 0.871$, in *n*-Ge $\Lambda(L) = 0.786$. If scattering by charged impurities is dominating, $\tau_0 = \tau_0^{(imp)}$ and $r = 3/2$. Then

$$\langle\tau^2\rangle/\langle\tau\rangle^2 = (3\pi/8)K_{7/2}(\eta)K_{1/2}(\eta)/[K_2(\eta)]^2. \quad (42)$$

If scattering on longitudinal acoustic phonons dominates, $\tau_0 = \tau_0^{(Ac)}$ and $r = -1/2$. Then

$$\langle\tau^2\rangle/\langle\tau\rangle^2 = (345\pi/512)K_{-1/2}(\eta)K_{1/2}(\eta)/[K_0(\eta)]^2. \quad (43)$$

One can see from Eqs (40) – (43) that the value $enc\, R_H(H \to 0)$ depends here on the mechanism of scattering and on dimensionless Fermi energy η. Intervalley drag in the framework of τ-approximation is evidently out of consideration.

Fig. 6 shows separating lines presented by the condition $\tau_0^{(imp)}/\tau_0^{(Ac)} = 1$. The line *1* relates to *n*-Si, the line *2* – *n*-Ge. Above the corresponding line $\tau_0^{(imp)}/\tau_0^{(Ac)} < 1$.

7. Discussion

Now compare results obtained for the method of balance equations and that of τ-approximation.

Solid lines in Fig. 7 show dependences of the value $enc\, R_H(H \to 0)$ on the energy η obtained by the τ-method for two different mechanisms of relaxation. These curves correspond to formulae (40), (42) and (43). The stroke lines $W^{(Si)}$ and $W^{(Ge)}$ in Fig. 7 represent the same value $enc\, R_H(H \to 0)$ but calculated using the method of balance equations (see Eqs (2)) applying the formulae (33), (34) and (35) (that is for the case $\xi = 0$,

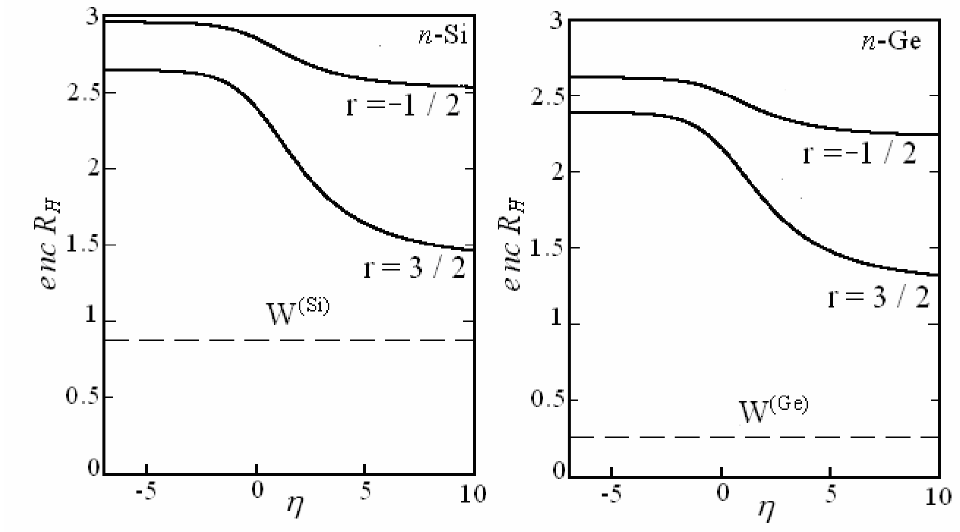

Fig. 7.

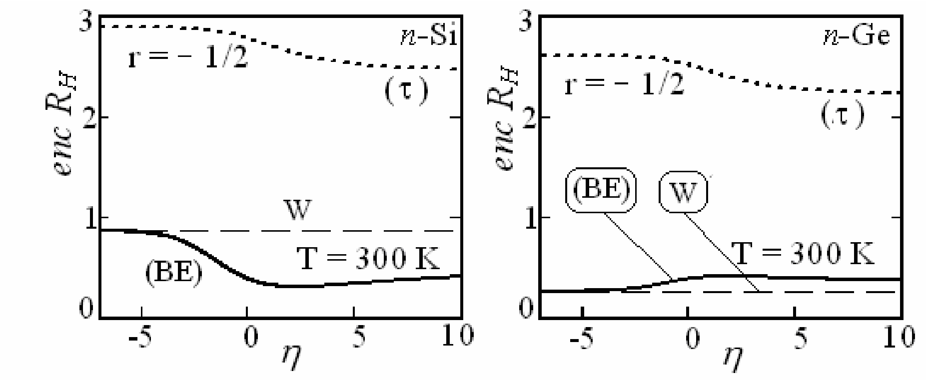

Fig . 8.

when intervalley drag is excluded from general consideration).

Fig. 8 relates to dominating contribution of acoustic phonons. Here, solid curves marked as BE represent results obtained using balance equations for the case when intervalley drag is involved in consideration ($\xi \neq 0$); the lines W correspond to the same way but for the case $\xi = 0$. Dashed curves marked by the letter τ represent solid lines in Fig. 7, which relate to the case $r = -1/2$.

One can see from Figs 7 and 8 that the difference between results obtained by two different methods of consideration is impressive.

References

1. I. I. Boiko, Electron-electron drag in crystals with many-valley band // *Semiconductor Physics, Quantum Electronics & Optoelectronics* **12** (3), p.212-217 (2009).
2. C. Herring, E. Vogt, Transport and deformation-potential theory for many-valley semiconductors with anisotropic scattering // *Phys. Rev*. **101,** p. 944 (1956).
3. A.I. Anselm *Introduction in Theory of Semiconductors*. Nauka, Moscow, 1978 (in Russian).
4. F. Gantmacher and I. B. Levinson: *Scattering of Current Carriers in Metals and Semiconductors*. Nauka, Moscow, 1975 (in Russian).
5. R. Smith, Semiconductors. Cambridge Univ. Press, 1978.
6. I. I. Boiko, *Kinetics of Electron Gas Interacting with Fluctuating Potential.* Naukova dumka, Kyiv, 1993 (in Russian).
7. I.I. Boiko, Transport of Carriers in Semiconductors. To be published, Kyiv, 2009 (in Russian).
8. I.I. Boiko. Specific thermo-emf in crystals with monopolar conductivity.// *Semiconductor Physics, Quantum Electronics & Optoelectronics* **12** (1), p. 47-52 (2009).
9. P.U. Yu, M. Cardona, F*undamentals of Semi-conductors. Physics and Material Properties*. Springer, 2002.