Sensors

Modeling of thermometric characteristics of thermodiode sensors by using the dimensionless sensitivity

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Abstract. Dimensionless sensitivity and slope of its characteristic in the forms $\alpha = d(\log V)/d(\log T)$ and $\gamma = d(\log\alpha)/d(\log T)$ have been proposed as a base for modeling of thermometric characteristics *V*(*T*). The differential analysis of *V*(*T*) curves within the range from 4.2 up to 400 K by numerical differentiation has allowed obtaining the analytical approximation in the form $V(T) = AT^{\alpha}\exp\left[-BT^{\gamma 1}\left(1+CT^{\gamma 2}\right)\right]$, where *A*, *B* and *C* are the constants depending on physical parameters of thermodiode silicon sensor. This approach is useful both for the analysis of these characteristics as well as for modeling and determining an approximating function by finding out the regions where power-like or exponential dependences are the adequate expressions to describe the thermometric characteristic sections. By contrast to the known methods, one should not know beforehand the function that describes the process or the characteristic. It permits to elucidate fine peculiarities of thermometric characteristics and to achieve high accuracy of modeling by using the analytical expressions. In view of the practical purposes, the thermometric characteristics are approximated within the three temperature ranges. The errors of approximation do not exceed ±0.02%, ±0.2% and ±0.4% within the temperature ranges 4.2…40 K, 40…170 K and 170…400 K, respectively.

Keywords: thermodiode sensor, thermometric characteristic, dimensionless sensitivity, modeling, approximation.

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1. Introduction

The thermometric characteristic (TMC) of temperature sensors, and thermodiode sensors in particular, is one of the most important ones [1, 2]. It defines the voltage (*V*) *vs* temperature (*T*) dependence at a constant measurement current value (*I*). This characteristic is necessary for the calibration of every device, and, thus, the accuracy of temperature measurements depends on the accuracy of thermometric characteristic. To make the process of thermometric characteristic determination more easy and cheap, it is possible to use a modeling method instead of direct measurements of this characteristic in the whole temperature range from liquid helium temperature to the room or higher one [3].

The available approaches to describe thermometric description can be conventionally separated by two large classes [4-8].

The methods of the first class are based on the mathematical formalism and are aimed at description of experimental curves by using various power series, polynomials or sets of exponents and logarithms, *e.g.*

$$V(T) = A + B\ln(T/T_1+1) + C[\ln(T/T_1+1)]^2 + \ldots, \quad (1)$$

$$V(T) = A_0 + A_1T + A_2 2T^2 + \ldots + A_mT^m + \ldots, \quad (2)$$

where *A*, A_0, A_1, A_2, A_m, *B* and *C* are the coefficients, T_1 is the characteristic temperature.

The methods of the second class are based on physical approximations, for example

$$V_f(T) = E_1 - E_2(a+T_r)(b+\ln T_r), \quad (3)$$

$$V(T) = E_0 - BT^2/(T+T_1) - CT\ln(DT), \quad (4)$$

where *a*, *b*, *B*, *C*, *D*, E_0, E_1, E_2 are the constants, $T_r = 1 + T/T_1$ is the reduced temperature.

However, these methods either do not provide necessary accuracy, or do not correlate with the physical principles of the device operation. Therefore, in the latter case the parameters of a semiconductor cannot be used in the approximations and, hence, in every case the modeling should be carried out anew.

In this paper, the modeling of thermometric characteristic of temperature sensors on the base of dimensionless sensitivity [9] is proposed for the first time. The dimensionless sensitivity was already used to characterize different types of temperature sensors [1, 10-13].

2. Principal assumptions of the differential method

The differential method for analyzing the $y(x)$ dependence, where x is the argument, y is the function, implies determination of dimensionless values

$$\alpha(x) = d(\lg y)/d(\lg x) = (dy/dx)/(y/x), \quad (5)$$

$$\gamma(x) = d(\lg \alpha)/d(\lg x) = (d\alpha/dx)/(\alpha/x). \quad (6)$$

The former value characterizes the power law dependence $y(x) = x^{\alpha}$, the latter one is the exponential law $y(x) = \exp(x^{\gamma})$. While processing the experimental dependences $y(x)$ by using the formula (5), one can reveal the regions with α = const. There are the regions where the $y(x)$ dependence is adequately approximated by a power law. Similarly, while processing experimental dependences $y(x)$ using (6) one will find the regions with γ = const. These regions correspond to almost exponential behaviour of $y(x)$.

The accuracy of $\alpha(x)$ and $\gamma(x)$ values is limited by the errors of the argument x and function $y(x)$ measurements, Δ_x and Δ_y, respectively. If the systematic error due to the transition from infinite steps to finite steps is taken into account, Δ_α and Δ_γ can be written like to those in [9]

$$\Delta_\alpha = 0.3\left(\left|\alpha^2 - 1\right|/\alpha\right)^{1/3}\left(\Delta_y + \alpha\Delta_x\right)^{2/3}, \quad (7)$$

$$\Delta_\gamma = 0.3\left(\left|\gamma^2 - 1\right|/\gamma\right)^{1/3}\left(\Delta_\alpha + \gamma\Delta_x\right)^{2/3}. \quad (8)$$

Application of the differential approach to the analysis of the thermometric characteristic by calculation $\alpha(T)$ and $\gamma(T)$ enables to elucidate the main features of its variation rate. From the physical point of view, $V(T)$ depends explicitly on $n(T)$, $p(T)$, $\mu_{n,p}(T)$, $\tau_{n,p}(T)$ and implicitly on $E_g(T)$, $\Delta E(t)$, $E_K(T)$, $\Delta E_A(T)$, $\varepsilon(T)$, $S_K(T)$, where n and p are the electrons and holes concentrations, respectively, $\mu_{n,p}$ and $\tau_{n,p}$ – mobility and the lifetime of electron and hole, respectively, E_g is the bandgap, ΔE – barrier height in the *p-n* junction, ΔE_K and ΔE_A are the barrier heights at the Me-*n* and Me-*p* boundaries, respectively, ε is the dielectric constant, S_K – rate of surface recombination. Moreover, various charge flow mechanisms reveal themselves with the temperature increase. They are: percolation conductivity, low-injection thermionic emission, tunneling of carriers, high-injection space-charge-limited regime and so on. There is a wide variety of temperature-dependent parameters that prevent analytical description of thermometric characteristics $V(T)$ in the wide temperature range [14, 15].

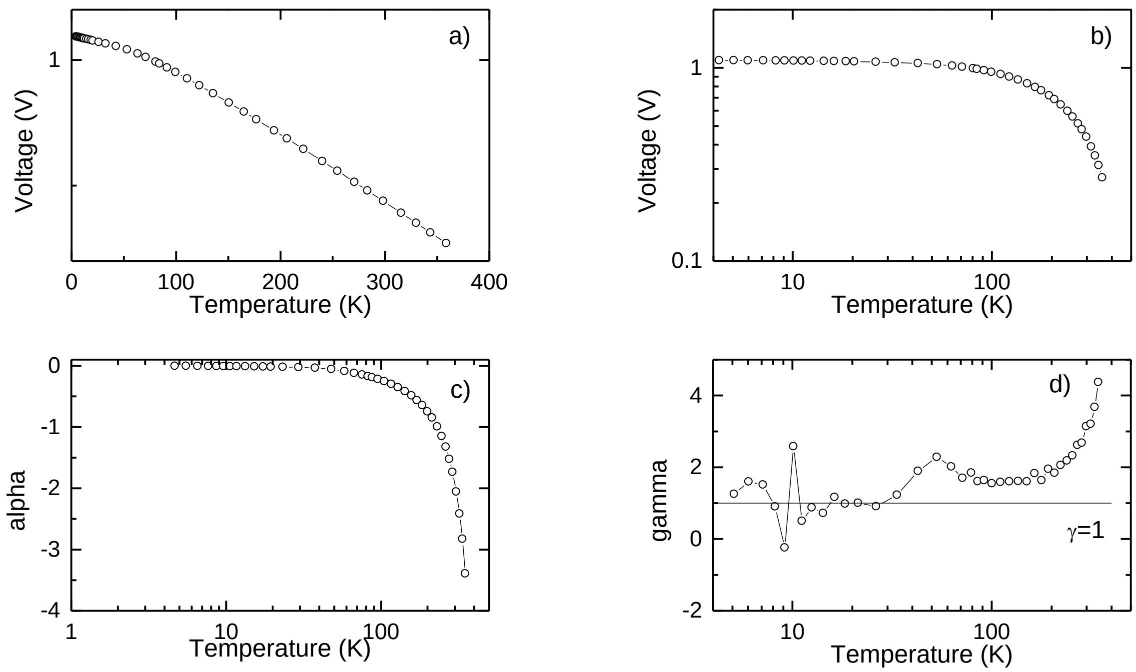

Fig. 1. Thermometric characteristic of Si diode of I type: a) TMC in the usual plot; b) TMC in the log-log plot; c) differential slope of TMC; d) slope of differential slope.

3. Experimental

Planar Si diodes of p^+-n type were experimentally investigated. The thermometric characteristics were measured within the temperature range 4.2…400 K using the exemplary equipment for temperature sensors graduation. The measurement current values were 1 μA and 10 μA; the accuracy of current stabilization was ±0.05 %. The error of temperature measurements was no more than 23 mK.

4. Thermometric characteristics and their modeling

The typical measured thermometric characteristics at the measurement current $I = 1$ μA (I type) and $I = 10$ μA (II type) are shown in Figs 1a, 1b and 2a, 2b.

One can see that it is very difficult to distinguish between the curves in Figs 1a, 1b and 2a, 2b. The analysis of the obtained TMC was carried out using the differential method in accordance with (5) and (6). The differential treatment of $V(T)$ exhibited the same features of $V(T)$ dependences at various measurement currents in general (Figs 1d and 2d).

The approximately constant value $\gamma = 1$ is typical for I type TMC (Fig. 1d) in the temperature range from 4.2 to 30 K. It is followed by the transition region with a slow increase of γ up to $\gamma = 2$ and consequent decrease down to $\gamma = 1.5$ within the range 30…80 K. Afterwards, γ increases starting from 150 K.

The II type TMC (Fig. 2d) exhibits the decrease of γ from 2.2 down to 1 in the initial temperature range 4.2…10 K, then γ increases from 1 up to 2.7 within the range 10…40 K, and after the transitional region 40…80 K γ saturates and becomes $\gamma = 1.5$ up to 200 K. The consequent increase of temperature leads to the increase of γ.

Thus, TMC behavior can be described at least by exponent-like dependences with $\gamma = 1$ in the low temperature region and $\gamma = 1.5$ in the relatively high one. And TMC behavior cannot be described by power-like dependence, because of there are no regions with $\alpha = \text{const}$. Unfortunately, the modeling by only one analytical expression in all the temperature range failed, because of insufficient accuracy of about 2%. To increase the accuracy of TMC for both types of thermodiode characteristics, the temperature range can be separated into three subranges. To our opinion, this separation is the result of changes in charge flow mechanisms. Nevertheless, the analytical expressions for all temperature ranges can be written as

$$V(T) = \frac{A\exp\left\{-(T/B)^{\gamma 1}\left[1-(T/C)^{\gamma 2}+(T/D)^{\gamma 3}\right]\right\}}{\left[\exp(E/T)^{\gamma 4}(E/T)^{\gamma 5}\right]^{n}} . \quad (9)$$

The corresponding coefficients are summarized in Table for each temperature range.

The complex character of approximations is caused by a lot of temperature parameters describing these TMCs as mentioned above. In Figs 3a and 4a, the ratio of calculated and measured TMCs is presented. One can see that the accuracy of calculated TMC is better than 0.2% in all the temperature range and than 0.04% within the range from 4.2 up to 100 K for the I type TMC.

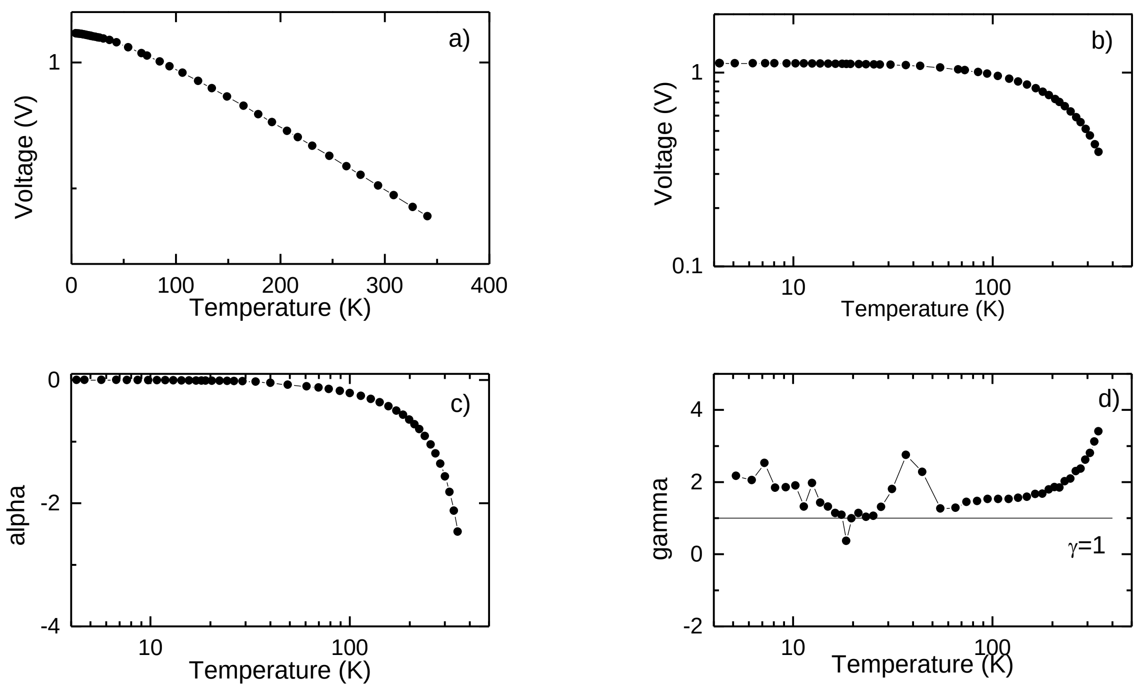

Fig. 2. Thermometric characteristic of Si diode of II type: a) TMC in the usual plot; b) TMC in the log-log plot; c) differential slope of TMC; d) slope of differential slope.

	I type	II type
Temperature range, K	4.2–40	4.2–30
Parameters	A = 1.0973, B = 1075, γ_1 = 1	A = 1.1202, B = 1300, D = 85, E = 1.03; γ_1 = 1, γ_3 = 5, γ_4 = 6, n = 0.6
Temperature range, K	40–170	30–180
Parameters	A = 1.12, B = 380, D = 420, E = 4.6; γ_1 = 1.45, γ_3 = 4, γ_4 = 2, n = 1	A = 1.1315, B = 370, D = 540, E = 3; γ_1 = 1.44, γ_3 = 4.7, γ_4 = 2, n = 1
Temperature range, K	170–400	120–400
Parameters	A = 1.1322, B = 350, C = 580, D = 444.8, E = 56; γ_1 = 1.45, γ_2 = 4, γ_3 = 5, γ_5 = 0.1, n = 1	A = 1.0984, B = 362, C = 590, D = 472, E = 56; γ_1 = 1.68, γ_2 = 4.7, γ_3 = 5, γ_5 = 0.0152, n = 1

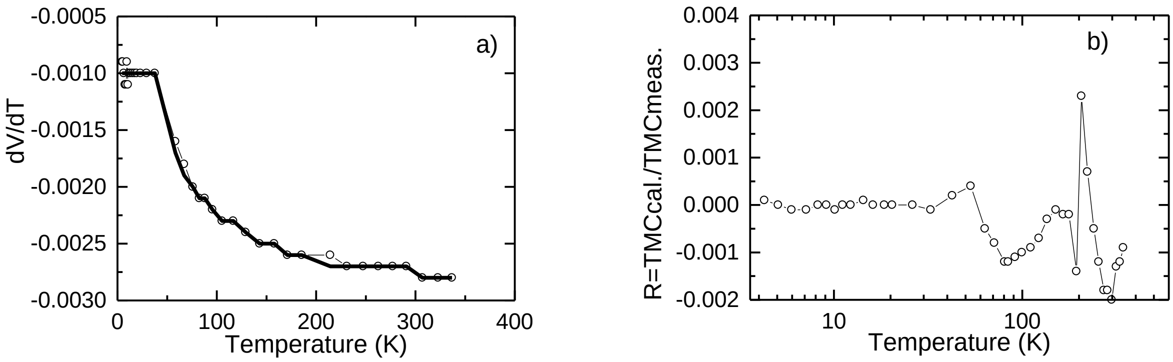

Fig. 3. Calculated (——) and measured (○) dV/dT curves (a) and their ratio R (b) for the I type of TMC.

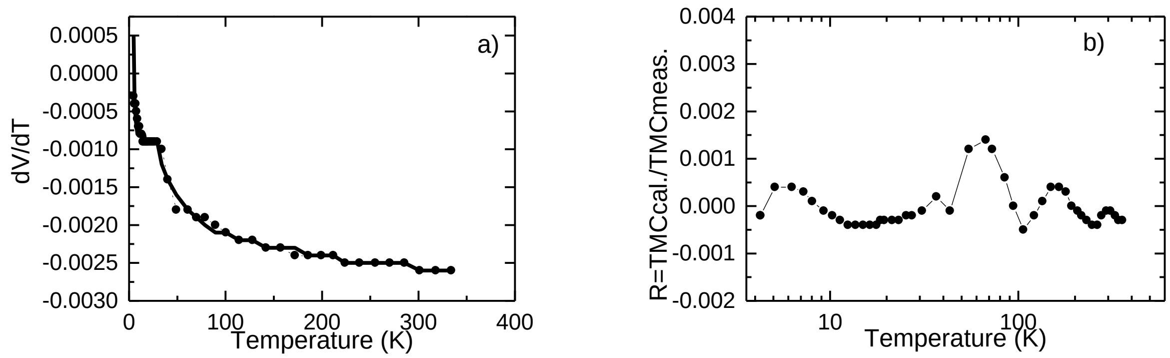

Fig. 4. Calculated (——) and measured (•) dV/dT curves (a) and their ratio R (b) for the II type of TMC.

5. Conclusions

The differential method can be offered as a basis for new approach to the diagnostics of thermoelectric characteristics. This approach is useful both for the analysis of characteristics and for determination of the approximating function by finding out the regions where power-like or exponential dependences are the adequate expressions for description of thermometric characteristic sections. By contrast to the known methods, one should not know beforehand the function that describes the process or characteristic.

The advantage of this method, as compared to the method of dimensionless temperature sensitivity, is determination of the slope of dimensionless sensitivity characteristic, which enables to elucidate fine peculiarities of thermometric characteristics and to achieve high accuracy of modeling using the analytical expressions.

In view of the practical purposes, the thermometric characteristics have been approximated in three temperature ranges. The errors of approximation do not exceed ±0.02%, ±0.2% and ±0.4% within the temperature ranges 4.2…40 K, 40…170 K and 170…400 K, respectively.

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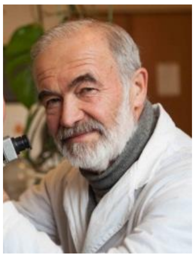

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Моделювання термометричних характеристик термодіодних датчиків з безрозмірною чутливістю

П.С. Смертенко

Анотація. Безрозмірна чутливість та нахил її характеристики у формі $\alpha = d(\log V)/d(\log T)$ та $\gamma = d(\log \alpha)/d(\log T)$ пропонуються як основа для моделювання термометричних характеристик *V*(*T*). За допомогою аналізу кривих *V*(*T*) у діапазоні від 4,2 K до 400 K методом чисельного диференціювання отримано аналітичне наближення у вигляді $V(T) = AT^{\alpha} \exp\left[-BT^{\gamma 1}\left(1 + CT^{\gamma 2}\right)\right]$, де *A*, *B* і *C* – константи, що залежать від фізичних параметрів термодіодного кремнієвого датчика. Цей підхід корисний як для аналізу цих характеристик, так і для моделювання та знаходження апроксимуючої функції шляхом визначення областей, де степеневі або експоненціальні залежності є адекватними виразами для опису окремих діапазонів термометричних характеристик. На відміну від відомих методів, не слід заздалегідь знати функцію, яка описує процес, або характеристику. Це дозволяє з'ясувати тонкі особливості термометричних характеристик та досягти високої точності моделювання за допомогою аналітичних виразів. З огляду на практичні цілі, термометричні характеристики наближені до трьох температурних діапазонів. Похибки апроксимації не перевищують 0,02%, 0,2% та 0,4% в межах температурних діапазонів 4,2… 40 K, 40… 170 K та 170… 400 K відповідно.

Ключові слова: термодіодний датчик, термометрична характеристика, безрозмірна чутливість, моделювання, апроксимація.