

# Diffusion length of non-equilibrium current carriers in nanowire radial $p$ - $n$ junctions: Effect of the curvature

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**Abstract.** In core-shell nanowire radial  $p$ - $n$  junction, spatial (along the radius) distribution of the injected carriers is determined not only by recombination falling of the non-equilibrium carrier concentration but also by specific falling due to cylindrical symmetry of the structure. This forces us to consider an effective diffusion length of non-equilibrium carriers in nanoscale radial structures. This effective diffusion length proves to be larger (up to 25%) than the diffusion length in usual planar  $p$ - $n$  junction (made of the same material) under injection from the shell to the core and smaller than it (up to 60%) under injection from the core to the shell.

**Keywords:** core-shell nanowire, radial  $p$ - $n$  junction, non-equilibrium current carriers, diffusion length.

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## 1. Introduction

Cylindrical symmetry of nanowire radial  $p$ - $n$  junction results in dependence of falling rate for the injected carriers' concentration on injection direction: out of the core to the shell or, *vice versa*, out of the shell to the core. Namely: the spatial falling rate of the carriers concentration proves to be larger than that in usual planar  $p$ - $n$  junction diode when carriers injecting out of the core to the shell and smaller than it when carrier injecting out of the shell to the core [1]. This effect is conditioned by superposition of the radial concentration falling connected with cylindrical symmetry of the structure onto the concentration falling caused by recombination of injected carriers.

In order to evaluate this effect quantitatively, it is worth to introduce a concept of effective diffusion length of non-equilibrium carriers in nanoscale radial  $p$ - $n$  structures, which is distinct from that in analogous planar diodes. These nanoscale radial  $p$ - $n$  structures are widely used now in variety of devices, namely: nanowire photo-detectors [2, 3], nanowire solar cells [3–5], nanowire light-emitting and laser diodes [6, 7], nanowires for energy harvesting [8], *etc.*

It should be noted that in such methods of energy harvesting as photovoltaics and betavoltaics, the diffusion length of non-equilibrium carriers determines also the capture length of excited carriers into region of built-in electric field of the  $p$ - $n$  junction.

Below two different cases of injection in the nanowire radial  $p$ - $n$  junction are considered in details.

## 2. Initial equations

The case of partially depleted  $p$ -core and  $n$ -shell is considered (Fig. 1). Here,  $r_0$  is the core radius,  $r_p$  indicates the depletion region boundary in the core,  $r_n$  corresponds to the depletion region boundary in the shell, and  $r_d$  is the external radius of the nanowire. For the sake of simplicity, surface depletion region is believed to be absent.

The real diffusion length of non-equilibrium current carriers can be assessed on spatial distribution of the carriers injected through  $p$ - $n$  junction. This distribution is determined by continuity equation  $\text{div}(D\text{grad}(u)) = u/\tau$ , where  $u$  is the excess concentration of minority carriers,  $D$  is their diffusion coefficient, and  $\tau$  is their lifetime. In the system of cylindrical coordinates this equation takes the form

$$D \left( \frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} \right) = \frac{u}{\tau}, \quad (1)$$

where  $r$  is a coordinate along the radius (axial and angular dependences are neglected). Replacement of the variable  $r/L = x$  where  $L = \sqrt{D\tau}$  is the standard diffusion length, gives the equation

$$\left( \frac{d^2 u}{dx^2} + \frac{1}{x} \frac{du}{dx} \right) = u. \quad (2)$$

Its solution, being expressed in terms of the fundamental system, is as follows:

$$u(x) = C_1 I_0(x) + C_2 K_0(x), \quad (3)$$

where  $I_0$  and  $K_0$  are modified Bessel's functions of 1<sup>st</sup> and 2<sup>nd</sup> kinds, respectively,  $C_1$  and  $C_2$  are the integration constants determined by the boundary conditions. The latter are the concentration of excess carriers provided by biasing of the  $p$ - $n$  junction and velocity of surface recombination.

### 3. Numerical calculations

#### 3.1. Injection out of the core to the shell (radial $p^+$ - $n$ diode)

In this case,  $u(x) = \Delta p(x)$  and solution of Eq. (3), in accordance with [1], has the form

$$\Delta p(x) = \Delta p_0(x_n) \times \frac{I_0(x) [K_1(x_d) - S^* K_0(x_d)] + K_0(x) [I_1(x_d) + S^* I_0(x_d)]}{I_0(x_n) [K_1(x_d) - S^* K_0(x_d)] + K_0(x_n) [I_1(x_d) + S^* I_0(x_d)]}, \quad (4)$$

where  $\Delta p_0(x_n)$  is the excess hole concentration at the depletion region boundary in the  $n$ -shell (emitter of holes) determined by the applied voltage,  $S^* = S L_p / D_p$  is the dimensionless surface recombination velocity at the nanowire external surface ( $L_p$  and  $D_p$  are the hole diffusion length and diffusion coefficient, respectively). All radial coordinates are normalized to the hole diffusion length  $L_p$ .

Fig. 2 presents this distribution for a number of the nanowire radii  $x_d$  at a fixed value of  $x_n$  for  $S^* = 0$  and  $S^* = \infty$ . As the measure of concentration falling, we will consider the distance  $L_e$  between  $x_n$  (as the source of injected carriers) and coordinate where the concentration decreases by  $e$  times (dash-dot line in the figure). It is seen that the distributions for  $S^* = 0$  and  $S^* = \infty$  are getting closer, when  $x_d$  increases going to the bulk one (*i.e.*, independent of the surface recombination).

The above characteristic distances  $L_e$  as functions of  $x_d$  are shown in Fig. 3 for a number of  $x_n$  values.

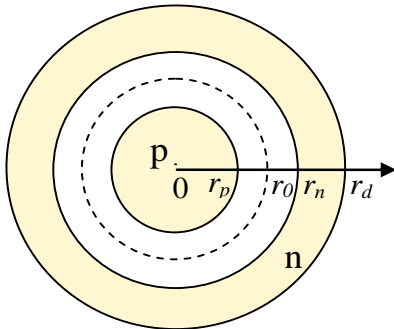


Fig. 1. Schematic view of the radial  $p$ - $n$  structure.

As it follows from these dependences, bulk values of the effective diffusion length (given by merging point of the curves for  $S^* = 0$  and  $S^* = \infty$ ) increase with  $x_n$  going to “planar” value  $L_p$  but staying substantially shorter than it (of the order of 0.4 to 0.8  $L_p$ ). *I.e.*, the effective diffusion length decreases with approaching the source of non-equilibrium carriers nearer to the nanowire axis.

#### 3.2. Injection out of the shell to the core (radial $p$ - $n^+$ diode)

In this case,  $u(x) = \Delta n(x)$  and solution of Eq. (3), in accordance with [1], is given by

$$\Delta n(x) = \Delta n_0(x_p) \times \frac{I_0(x) [K_1(0) - S^* K_0(0)] + K_0(x) [I_1(0) + S^* I_0(0)]}{I_0(x_p) [K_1(0) - S^* K_0(0)] + K_0(x_p) [I_1(0) + S^* I_0(0)]}, \quad (5)$$

where all the radial coordinates are normalized to the electron diffusion length  $L_n$ ,  $\Delta n_0(x_p)$  is the excess electron concentration at the depletion region boundary  $x_p$  in the  $p$ -core (emitter of electrons) determined by the applied voltage,  $S^*$  is dimensionless surface recombination velocity at nanowire axis. In reality, the central contact is located at the bottom of the nanowire. Therefore, for the nanowire cross-sections far from the bottom, the surface recombination velocity at  $x = 0$  may be put equal to zero, but for the cross-sections near the bottom, it may be arbitrary.

Distribution (5) is presented in Fig. 4 at a number of  $x_p$  values for  $S^* = 0$  and  $S^* = \infty$  (it is worth-while to note that the diffusion lengths can be very small in reality, therefore  $x_p = 5$  is quite possible).

At sufficiently large  $x_p$ , the curves for  $S^* = 0$  and  $S^* = \infty$  are merging; that indicates independence of

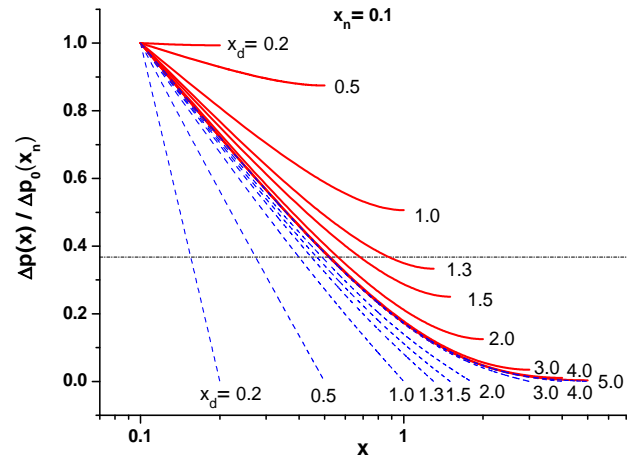
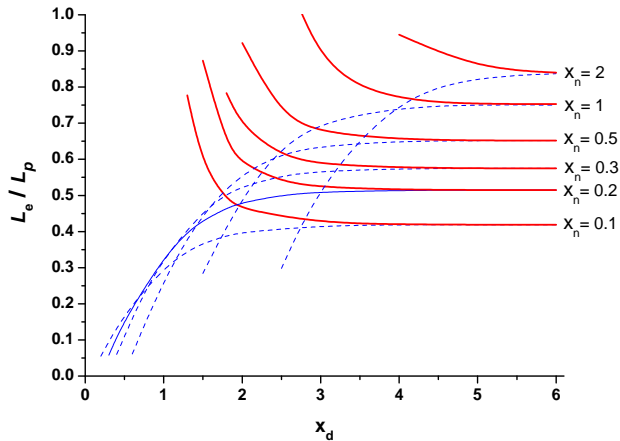
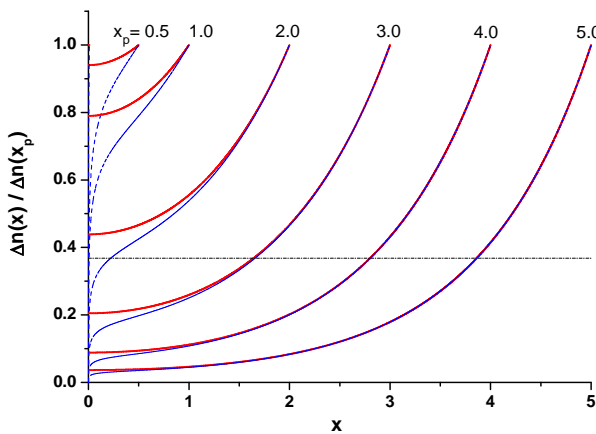


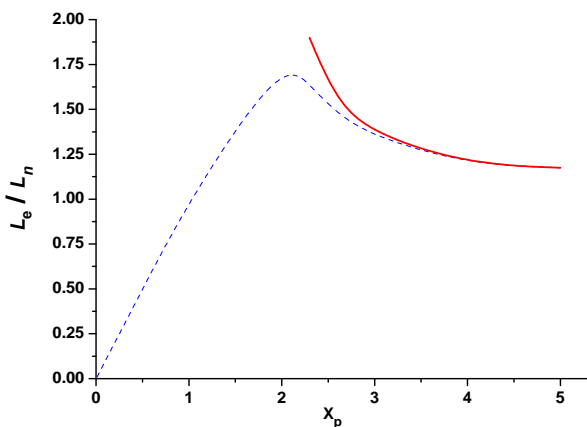
Fig. 2. Distribution of injected carriers in the base of radial  $p^+$ - $n$  diode depending on the nanowire radius  $x_d$  as a parameter at  $S^* = 0$  (solid lines) and  $S^* = \infty$  (dash lines).



**Fig. 3.** Characteristic lengths of the concentration falling in the base of radial  $p^+n$  diode as a function of the nanowire radius  $x_d$  at different locations of injection point  $x_n$  for  $S^* = 0$  (solid lines) and  $S^* = \infty$  (dash ones).



**Fig. 4.** Distributions of injected carriers in the base of radial  $p-n^+$  diode depending on injection point  $x_p$  as a parameter at  $S^* = 0$  (solid lines) and  $S^* = \infty$  (dash lines).



**Fig. 5.** Characteristic lengths of concentration falling in the base of radial  $p-n^+$  diode versus the coordinate of injection point  $x_p$  for  $S^* = 0$  (solid lines) and  $S^* = \infty$  (dash ones).

the concentration distribution on surface recombination, *i.e.* bulk properties. As the measure of the concentration falling, we will consider here the distance  $L_e$  between  $x_p$  (as the source of injected carriers) and the coordinate, where the concentration is  $e$  times decreased (dash-dot line in the figure).

Dependences of those characteristic lengths  $L_e$  on location of the injection point  $x_p$  are shown in Fig. 5. It is seen that at  $x_p \geq 3$ , the curves for  $S^* = 0$  and  $S^* = \infty$  are merging, which gives the bulk  $L_e$  value. This value proves to be substantially larger in this case than the “planar” diffusion length (of the order of  $1.25L_n$ ).

#### 4. Conclusion

Thus, the effective diffusion length of non-equilibrium carriers in radial core-shell  $p-n$  junction differs substantially from that in planar diode and depends on in what direction the diffusion proceeds – from the nanowire center to periphery or, *vice versa*, from periphery to the center. In particular, it means that the capture length of excited carriers into region of the built-in electric field of the  $p-n$  junction (in radial core-shell photodetectors and solar cells as well as in radial betavoltaic batteries) proves to be smaller in the core (because diffusion is directed from the nanowire center) and larger in the shell (where diffusion is directed to the center). *I.e.*, the radial capture of the non-equilibrium carriers into the field region is substantially asymmetrical in this case. And, in general, all size-sensitive effects, *i.e.* effects when characteristic size of the object equates to the diffusion length of carriers, undergo changes under cylindrical symmetry.

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#### Дифузійна довжина нерівноважних носіїв струму в нанодрових радіальних $p-n$ переходах: роль кривизни

##### В.Л. Борблик

**Анотація.** У нанодрових типу ядро-оболонка радіальних  $p-n$  переходах просторовий (вздовж радіуса) розподіл інжекттованих носіїв визначається не тільки рекомбінаційним спаданням концентрації нерівноважних носіїв, але також специфічним спаданням, зумовленим циліндричною симетрією структури. Це змушує нас говорити про ефективну дифузійну довжину нерівноважних носіїв у нанорозмірних радіальних структурах. Така ефективна дифузійна довжина виявляється більшою (до 25%), ніж дифузійна довжина у звичайних планарних  $p-n$  переходах (виготовлених з того самого матеріалу) у випадку інжекції з оболонки в ядро і меншою за неї (до 60%) при інжекції з ядра в оболонку.

**Ключові слова:** нанодріт типу ядро-оболонка, радіальний  $p-n$  перехід, нерівноважні носії струму, дифузійна довжина.