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# Residual error after non-uniformity correction

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**Abstract.** The paper represents the technique for residual error evaluation after two-points linear non-uniformity correction. This technique takes into consideration parameters of an imaging system, reference sources, non-linearity and noise of a focal plane array.

**Keywords:** non-uniformity correction, photosensitive element, reference source.

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## 1. Introduction

Modern infrared imaging systems for remote sensing and temperature distribution measurement apply focal plane arrays. But the photosensitive elements of a focal plane array (FPA) have different properties, in particular their responsibilities and dark currents are not uniform. It reduces performance of infrared imaging systems and as result accuracy of measurements of temperature distribution is getting lower [1, 2, 4]. To avoid the influence of non-uniformity of photosensitive elements it is necessary to apply the special procedures for non-uniformity correction [1, 2]. These procedures use output signals from two or more reference sources to calculate the coefficients for the non-uniformity correction and to restore the output signal [2]. The most common approach is application of two reference sources that have the temperatures $T_1$ and $T_2$ respectively with linear interpolation the output signal. It is called linear two-point non-uniformity correction (NUC) [1]. Generally the temperatures $T_1$ and $T_2$ are defined by the limits of working temperature range [1].

## 2. Two-point linear correction

The linear interpolation of the output signal can be written in the form:

$$U_{\text{nuc}} = K \cdot \frac{U - U_1}{U_2 - U_1} + U_f \tag{1}$$

where $U_{\text{NUC}}$ is an output signal of the photosensitive element (PSE) after NUC; $U$ is an output signal of the PSE; $U_1, U_2$ are output signals of the PSE from the reference sources, generally blackbodies with the temperatures $T_1$ and $T_2$, respectively; $K$ is a coefficient of amplification of the NUC unit, $U_f$ is an offset introduced by the NUC unit.

The output signals $U$, $U_1$, $U_2$ contain several parts such as the photo-signal, the dark current signal and noise. The function that links irradiance of PSE with PSE output signal called signal transfer function is not linear. So it has to introduce some non-linearity, for example, by an addition of the square term. Summarizing said above, the expression for calculation will have the following form (1):

$$\begin{aligned}
U &= R \cdot A \cdot \left(E - \xi \cdot E^2\right) + U_d + U_n = \\
&= R \cdot A \cdot \left(k \cdot M - \xi \cdot k^2 \cdot M^2\right) + U_d + U_n = \\
&= R \cdot A \cdot \left(k \cdot M(T) - \xi \cdot k^2 \cdot M^2(T)\right) + U_d + U_n \\
U_1 &= R \cdot A \cdot \left(E_1 - \xi \cdot {E_1}^2\right) + U_{d1} + U_{n1} = \\
&= R \cdot A \cdot \left(k_1 \cdot M_1 - \xi \cdot {k_1}^2 \cdot {M_1}^2\right) + U_{d1} + U_{n1} = \\
&= R \cdot A \cdot \left(k_1 \cdot M(T_1) - \xi \cdot {k_1}^2 \cdot M^2(T_1)\right) + U_{d1} + U_{n1} \\
U_2 &= R \cdot A \cdot \left(E_2 - \xi \cdot {E_2}^2\right) + U_{d2} + U_{n2} = \\
&= R \cdot A \cdot \left(k_2 \cdot M - \xi \cdot {k_2}^2 \cdot {M_2}^2\right) + U_{d2} + U_{n2} = \\
&= R \cdot A \cdot \left(k_2 \cdot M(T_2) - \xi \cdot {k_2}^2 \cdot M^2(T_2)\right) + U_{d2} + U_{n2}
\end{aligned} \tag{2}$$

where $R$, $A$ are the sensitivity and area of the PSE, respectively; $E$, $E_1$, $E_2$ are irradiances generated by the target and by the reference sources with the temperatures $T_1$ and $T_2$, respectively, $M$, $M_1$, $M_2$ are exitance of the target and the reference sources with the temperatures $T_1$ and $T_2$, respectively, $\xi$ is a coefficient of non-linearity of irradiance-to-signal transformation, $U_d$, $U_{d1}$, $U_{d2}$ are dark currents generated during exposition by image and by the reference sources with the temperatures $T_1$ and $T_2$, respectively, $U_n$ , $U_{n1}$ , $U_{n2}$ are noise generated during exposition by image and by the reference sources with the temperatures $T_1$ and $T_2$, respectively; $k$, $k_1$, $k_2$ are coefficients that link the exitance of target and the reference sources with the temperatures $T_1$ and $T_2$ is the course of PSE irradiation, respectively. These coefficients can be calculated according to schemes of PSE illumination taking into account spectral transmittance of optics, spectral responsibility of FPA, signal integration and geometric configuration of reference source scheme. The known techniques allows calculating the coefficients [4].

## 3. Sources of residual error

It is obvious that some factors like noise and variance of reference source temperatures distort the result of the NUC. We suppose that noise and variance of reference source temperatures are non-correlated so we can write the residual error after NUC in the following form:

$$\delta U_{nuc} = \left\{ \left( \frac{\partial U_{nuc}}{\partial U} \cdot \delta U \right)^2 + \left( \frac{\partial U_{nuc}}{\partial U_1} \cdot \delta U_1 \right)^2 + \left( \frac{\partial U_{nuc}}{\partial U_2} \cdot \delta U_2 \right)^2 \right\}^{1/2} =$$
$$= \left\{ \left( \left( \frac{\partial U_{nuc}}{\partial U} \cdot \frac{\partial U}{\partial M} \cdot \frac{\partial M}{\partial T} \cdot \delta T \right)^2 + \delta U_n^2 \right) + \left( \left( \frac{\partial U_{nuc}}{\partial U_1} \cdot \frac{\partial U_1}{\partial M_1} \cdot \frac{\partial M_1}{\partial T_1} \cdot \delta T_1 \right)^2 + \delta U_{n1}^2 \right) + \left( \left( \frac{\partial U_{nuc}}{\partial U_2} \cdot \frac{\partial U_2}{\partial M_2} \cdot \frac{\partial M_2}{\partial T_2} \cdot \delta T_2 \right)^2 + \delta U_{n2}^2 \right) \right\}^{1/2} \quad (3)$$

where $\delta U_{NUC}$ is a residual error after NUC as the variance of $U_{NUC}$; $\delta U$, $\delta U_1$, $\delta U_2$ are variances of the signal $U$, $U_1$, $U_2$, respectively. They are caused by two principal factors – noise and variances of reference source temperature; $\delta U_n$ , $\delta U_{n1}$ , $\delta U_{n2}$ are variances of the noise corresponding to the signals $U$, $U_1$, $U_2$, respectively. Let us suppose that these variances are proportional to the noise equivalent temperature differences NETD , $\mathrm{NETD}_1$ and $\mathrm{NETD}_2$ of an infrared imaging system:

$$\delta U_n = R \cdot A \cdot k \cdot \frac{\partial M(T)}{\partial T} \cdot \mathrm{NETD}$$
$$\delta U_{n1} = R \cdot A \cdot k_1 \cdot \frac{\partial M(T|T = T_1)}{\partial T} \cdot \mathrm{NETD}_1 \quad (4)$$
$$\delta U_{n2} = R \cdot A \cdot k_2 \cdot \frac{\partial M(T|T = T_2)}{\partial T} \cdot \mathrm{NETD}_2$$

$\partial U_{NUC}/\partial U$, $\partial U_{NUC}/\partial U_1$, $\partial U_{NUC}/\partial U_2$ are derivatives that represent the weights of influences of $U$, $U_1$, $U_2$ to $U_{NUC}$ (2):

$$\frac{\partial U_{nuc}}{\partial U} = \frac{k}{U_2 - U_1}$$
$$\frac{\partial U_{nuc}}{\partial U_1} = \frac{k}{U_2 - U_1} \cdot \left( \frac{U - U_1}{U_2 - U_1} - 1 \right) \quad (5)$$
$$\frac{\partial U_{nuc}}{\partial U_2} = \frac{k}{U_2 - U_1} \cdot \left( -\frac{U - U_1}{U_2 - U_1} \right)$$

$\delta U$, $\delta U_1$, $\delta U_2$ are variances of the output signal of the PSE caused by the variation of target temperature and by the variation of reference sources temperatures $T_1$ and $T_2$, respectively; $\partial U/\partial M$, $\partial U_1/\partial M_1$, $\partial U_2/\partial M_2$ are derivatives that represent the weights of influences of $M$, $M_1$, $M_2$ to $U$, $U_1$, $U_2$, respectively (2):

$$\frac{\partial U}{\partial M} = R \cdot A \cdot (1 - 2 \cdot \xi \cdot k \cdot M)$$
$$\frac{\partial U_1}{\partial M_1} = R \cdot A \cdot (1 - 2 \cdot \xi \cdot k_1 \cdot M_1) \quad (6)$$
$$\frac{\partial U_2}{\partial M_2} = R \cdot A \cdot (1 - 2 \cdot \xi \cdot k_2 \cdot M_2)$$

$M(T)$, $\partial M/\partial T$ are relationship between the exitance and the temperature of the target or the reference source and its derivative, respectively. We suppose that the Planck formula describes these relationships:

$$M(T) = \int_{\lambda_1}^{\lambda_2} M(\lambda, T) \cdot d\lambda = \int_{\lambda_1}^{\lambda_2} \frac{c_1}{\lambda^5 \cdot \left( \exp\left( \frac{c_2}{\lambda \cdot T} \right) - 1 \right)} \cdot d\lambda$$
$$\frac{\partial M(T)}{\partial T} = \int_{\lambda_1}^{\lambda_2} \frac{\partial M(\lambda, T)}{\partial T} \cdot d\lambda = \int_{\lambda_1}^{\lambda_2} \frac{c_1 \cdot c_2 \cdot \exp\left( \frac{c_2}{\lambda \cdot T} \right)}{\lambda^6 \cdot T^3 \cdot \left( \exp\left( \frac{c_2}{\lambda \cdot T} \right) - 1 \right)^2} \cdot d\lambda \quad (7)$$

where $l$ is a wavelength, $l_1$ to $l_2$ is a spectral range, $c_1$ , $c_2$ are constants [3,4].

$$\delta U_{nuc} = \frac{K}{(k_2 \cdot M(T_2) - k_1 \cdot M(T_1)) - \xi \cdot \left(k_2{}^2 \cdot M^2(T_2) - k_2{}^2 \cdot M^2(T_1)\right)} \times \left\{ \left[ \left( k \cdot (1 - 2 \cdot \xi \cdot M(T)) \cdot \left( \frac{\partial M(T)}{\partial T} \right) \right)^2 \times \right.\right.$$
$$\times \left(\delta T^2 + NETD^2\right)\Bigg] + \left( \frac{(k \cdot M(T) - k_1 \cdot M(T_1)) - \xi \cdot \left(k^2 \cdot M^2(T) - k_2{}^2 \cdot M^2(T_1)\right)}{(k_2 \cdot M(T_2) - k_1 \cdot M(T_1)) - \xi \cdot \left(k_2{}^2 \cdot M^2(T_2) - k_2{}^2 \cdot M^2(T_1)\right)} - 1 \right)^2 \times$$
$$\times \left[ \left( k_1 \cdot (1 - 2 \cdot \xi \cdot M(T_1)) \cdot \left( \frac{\partial M(T|T = T_1)}{\partial T} \right) \right)^2 \times \left(\delta T_1{}^2 + NETD_1^2\right) \right] +$$
$$+ \left( - \frac{(k \cdot M(T) - k_1 \cdot M(T_1)) - \xi \cdot \left(k^2 \cdot M^2(T) - k_2{}^2 \cdot M^2(T_1)\right)}{(k_2 \cdot M(T_2) - k_1 \cdot M(T_1)) - \xi \cdot \left(k_2{}^2 \cdot M^2(T_2) - k_2{}^2 \cdot M^2(T_1)\right)} \right)^2 \times$$
$$\left. \times \left[ \left( k_2 \cdot (1 - 2 \cdot \xi \cdot M(T_2)) \cdot \left( \frac{\partial M(T|T = T_2)}{\partial T} \right) \right)^2 \times \left(\delta T_2{}^2 + NETD_2^2\right) \right] \right\} \quad (8)$$

## 4. Residual error evaluation procedure

To get the final expression that binds noise, temperatures of the references sources and their uncertainties, a temperature of the target and its uncertainty with the residual error we put formulae (2, 4, 5, 6) into (3):

The expression (8) describes the influence of the most principal factors to the residual error such as non-linearity, noise and uncertainty. In the case of linear signal transfer function ($x = 0$) the expression (8) becomes more simple:

$$\delta U_{nuc} = \frac{K}{(k_2 \cdot M(T_2) - k_1 \cdot M(T_1))} \times$$
$$\times \left\{ \left[ \left( k \cdot \frac{\partial M(T)}{\partial T} \right)^2 \cdot \left(\delta T^2 + NETD^2\right) \right] + \right.$$
$$+ \left( \frac{(k \cdot M(T) - k_1 \cdot M(T_1))}{(k_2 \cdot M(T_2) - k_1 \cdot M(T_1))} - 1 \right)^2 \times$$
$$\times \left[ \left( k_1 \cdot \frac{\partial M(T|T = T_1)}{\partial T} \right)^2 \cdot \left(\delta T_1{}^2 + NETD_1^2\right) \right] +$$
$$+ \left( - \frac{(k \cdot M(T) - k_1 \cdot M(T_1))}{(k_2 \cdot M(T_2) - k_1 \cdot M(T_1))} \right)^2 \times$$
$$\left. \times \left[ \left( k_2 \cdot \frac{\partial M(T|T = T_2)}{\partial T} \right)^2 \cdot \left(\delta T_2{}^2 + NETD_2^2\right) \right] \right\}^{1/2} \quad (9)$$

When the noise is neglected - NETD=0 - the formula (9) will have the form similar to the mentioned error after NUC [1]:

$$\delta U_{nuc} = \frac{K}{(k_2 \cdot M(T_2) - k_1 \cdot M(T_1))} \times$$
$$\times \left\{ \left[ \left( k \cdot \frac{\partial M(T)}{\partial T} \right)^2 \cdot \delta T^2 \right] + \right.$$
$$+ \left( \frac{(k \cdot M(T) - k_1 \cdot M(T_1))}{(k_2 \cdot M(T_2) - k_1 \cdot M(T_1))} - 1 \right)^2 \times$$
$$\times \left[ \left( k_1 \cdot \frac{\partial M(T|T = T_1)}{\partial T} \right)^2 \cdot \delta T_1{}^2 \right] +$$
$$+ \left( - \frac{(k \cdot M(T) - k_1 \cdot M(T_1))}{(k_2 \cdot M(T_2) - k_1 \cdot M(T_1))} \right)^2 \times$$
$$\left. \times \left[ \left( k_2 \cdot \frac{\partial M(T|T = T_2)}{\partial T} \right)^2 \cdot \delta T_2{}^2 \right] \right\}^{1/2} \quad (10)$$

## 5. Results of computer simulation

The proposed technique for residual error evaluation was applied for analysis of the scanning infrared imaging system (Table 1). The aim of the analysis was to determine the influence of the noise, non-linearity and

**Table 1. Parameters of the scanning infrared imaging system.**

| № | Parameter | Value |
|---|---|---|
| 1 | Focal length | 150 mm |
| 2 | Aperture diameter | 100 mm |
| 3 | Dimensions of PSE | 50x50 µm |
| 4 | Energy loss | 0.35 |
| 5 | Time of signal integration | 25 µsec |
| 6 | Quantum efficiency | 0.85 |
| 7 | Background temperature | 283 K |
| 8 | Temperature $T_1$ | 293 K |
| 9 | Temperature $T_2$ | 333 K |
| 10 | Wavelength range | 8–12 µm |

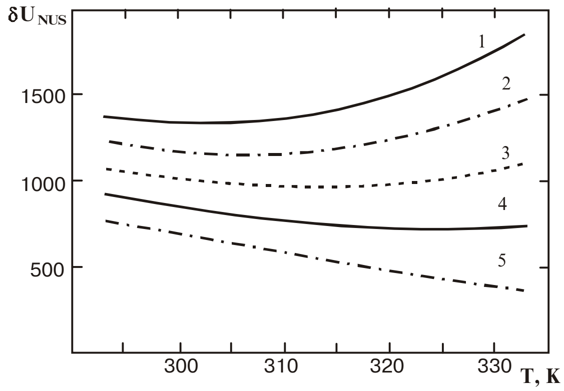


**Fig. 1**. Residual error $dU_{NUC}$ in a number of electrons as a function of the object temperature T for different values of non-linearity, $\xi$: 1 - 0 , 2 - 0.1 , 3 - 0.2 , 4 - 0.3, 5 - 0.4; $\delta T = 0$ K , $\delta T_1 = \delta T_2 = 0$ K; $K = U_2 - U_1$.

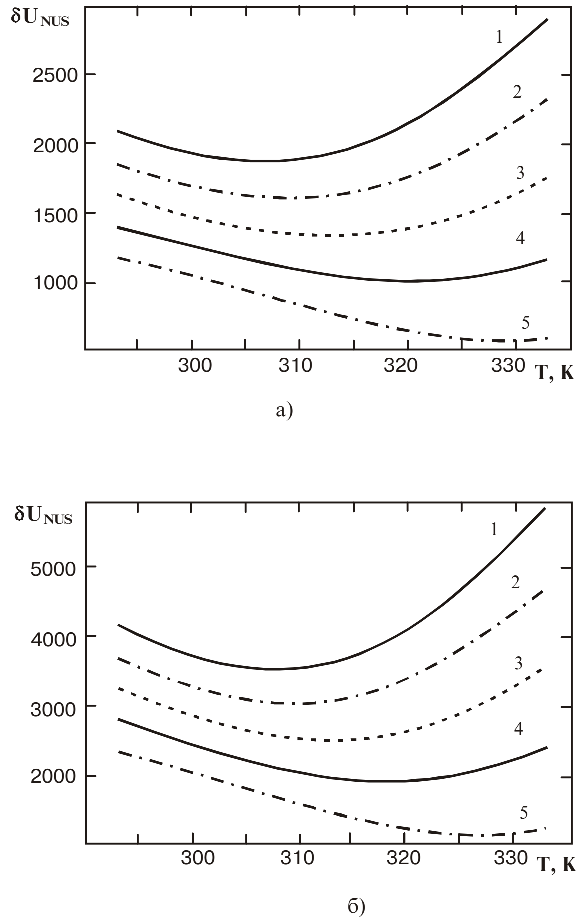


**Fig. 2.** Residual error $dU_{NUC}$ in a number of electrons as a function of the object's temperature T for different values of non-linearity:1 - $\xi = 0$ , 2 - $\xi = 0.1$, 3 - $\xi = 0.2$ , 4 - $\xi = 0.3$ , 5 - $\xi = 0.4$; a) $\delta T = 0$ K, $\delta T_1 = \delta T_2 = 0.10$ K ; b) $\delta T = 0$ K , $\delta T_1 = \delta T_2 = 0.25$ K ($K = U_2 - U_1$).

variances of reference sources temperatures on the residual error. The value of NETD was calculated for a photon-noise limited imaging system [3,4]. The results of computer simulation show the following facts that had not been pointed out early (Figs 1, 2) [1, 2]. First, non-linearity dramatically reduces the residual error because a saturation limits the variance between output signals from different photo-sensitive elements. Second, the shape of residual error function is not symmetrical because the bigger object's temperature increases also photon noise which makes the residual error bigger (Figs 1, 2). Third, residual error and noise are interconnected: in an infrared imaging system a residual error is a result of influence of noise, but the noise contains several parts including spatial-fixed noise caused by non-uniformity. In other words, it has to consider a residual error and a noise as a common property of imaging systems but not as separate phenomena.

## Conclusions

The proposed mathematical apparatus makes it possible to evaluate of a residual error after NUC as function of noise, variance of reference source temperatures and non-linearity. The expression (8) represents the common case when all principal factors act together. Simplifying the expression (8) we can obtain the formula (10) that is a particular case mentioned in the papers [1,2].

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